Prudential Regulation and Bank Accounting

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Abstract

This study focuses on the interaction between prudential regulation and accounting standard setting for banks in a context of banks' asset substitution decision and quality choice of loan portfolios. The bank regulator sets the regulatory leverages for banks, and the accounting standards require banks to use either fair value accounting or historical cost accounting to report on loan performance. Using the ex ante bank value as the criterion, we find that the historical cost regime dominates the fair value regime for medium values of asset substitution opportunity; medium values of asset substitution constraint; low values of asset specificity; low values of fundamental risk of loan portfolios; and low values of the liquidity benefit of bank debtholders. Fair value accounting dominates for other values of these parameters. This study contributes to the theoretical literature on accounting for banks by incorporating *both the asset side and the liability side* of the banks' balance sheets in order to shed light on the debate about the bank opacity, and by introducing regulatory coordinations, that is, how *prudential regulation and accounting standard setting* are optimally coordinated.

Keywords: regulatory leverage; fair value accounting; historical cost accounting; loan quality; asset substitution.

1 Introduction

Debates are ongoing on alternative financial accounting standards for banks in the context of prudential regulation of banks. Some argue for the importance of regulating the *assets* side of the banks' balance sheet. For example, Morgan (2002, p. 874) states that "the opacity of banks exposes the entire financial system to bank runs, contagion, and other strains of 'systemic' risk. Take away opacity and the whole story unravels." Similarly, Nier and Bauman (2006, p. 337) believe that "a bank that discloses its risk profile exposes itself to market discipline and will therefore be penalized by investors for choosing higher risk." Therefore, this type of views support *fair value* accounting, which reports timely interim performance of banks' loan portfolios, and thereby making it feasible for the prudential regulators to tie the regulatory target leverages to the timely accounting information, and making it possible for the regulators to fine tune the regulatory leverages in order to precisely control the banks' asset substitution decisions.

However, others argue for the importance of the *liabilities* side of the banks' balance sheet. They emphasize the role of banks in the economy in creating highly liquid, money-like debt claims (such as demand deposits and the associated banking services). Such claims are collateralized to make them information-insensitive. To further make them information-insensitive, they argue, banks should be "secret keepers," and thus governmental guarantees, and regulation and supervision should not force banks to publicly reveal information (Dang et al. 2014; Holmstrom 2015). Therefore, this type of views support *historical cost* accounting, which does not report timely interim performance of banks' loan portfolios, and thereby making the banks' public disclosure and thus the regulatory target leverages information-insensitive, which enhances the debtholders' liquidity benefits.

We investigate how the prudential regulations and accounting standards should be optimally coordinated to enhance the ex ante bank value (the sum of the bank's ex ante debt value and equity value). Specifically, how will the optimal regulatory leverages vary with different accounting regimes (fair value accounting versus historical cost accounting)? Given such optimal regulatory leverages, under what conditions will one accounting regime dominate the other?

Banks are plagued with asset substitution problems, in which the bank increases the risk of its loan portfolios to gamble for the upside potential at the expense of the debtholders. For example, they may roll over nonperforming loans or undertake negative net present value projects (Jensen and Meckling 1976; Dewatripont and Tirole 1993; Gron and Winton 2001), or they may double up bets on illiquid securities that have experienced significant adverse price shocks.[1] Banks are also subject to debt overhang problems, in which excessive leverage induces banks to forgo positive net present value projects and thus lead to underinvestment in the quality of the loan portfolios (Myers 1977; Admati et al. 2012).

We incorporate these two prevalent bank problems in a two-period model, in which the bank chooses the quality of its loan portfolios in period 1 and makes the asset substitution decision in period 2. Asset substitution in period 2 may enhance the bank's equity value in period 2; anticipating this, the bank is incentivized to choose a higher quality level for its loan portfolios in period 1. Put it the other way, if the bank's period 2 asset substitution is constrained, its period 1 incentive for quality will be dampened (Lu, Sapra, and Subramanian 2018).

Apparently, the higher the bank leverage, the more incentivized the bank will be to engage in asset substitution. Thus, it may seem naturally that the regulator may want to lower the regulatory target leverage in period 2 to constrain asset substitution. However, this will reduce the bank's incentive for quality, as discussed above. To counteract this and induce a high quality, the regulator may lower the regulatory target leverage in period 1 to reduce the debt overhang on the bank. If so, the regulatory target leverages are lowered and thus the bank's debt capacity is reduced, which implies lower liquidity benefits for the debtholders.

We capture such economic tradeoffs in our model, and more important, we introduce an accounting tradeoff (fair value accounting versus historical cost accounting as mentioned above). Thus, we investigate the real effects of accounting for banks. Specifically, the main

[1] Examples for the former is the U.S. banks' lending to less-developed countries in the 1980s and Japanese banks' lending to zombie borrowers in the early 1990s, and an example for the latter is the U.S. thrifts' accumulation of junk bonds and other risky assets in the 1980s (Archaya and Ryan 2016).

ingredients of the model are (1) banks' asset substitution and quality of loan portfolios (the assets side of the bank's balance sheet) and (2) the bank debtholders liquidity benefit (the liabilities side of the balance sheet), (3) the regulatory target leverages for banks (prudential regulation), and (4) historical cost accounting versus fair value accounting (accounting standard setting).

Because both bank assets and bank liabilities are important for bank value, we focus on the following parameters of bank assets and liabilities: (1) asset substitution opportunity and constraint; (2) asset specificity (or more generally, the bank's debt capacity); (3) fundamental risk of loan portfolios; and (4) the liquidity benefit of bank debtholders.

Our main results are on the conditions under which the historical cost accounting regime dominates the fair value accounting regime and the conditions under which the converse is true, using the ex ante bank value as the criterion. we find that the historical cost regime dominates the fair value regime for medium values of asset substitution opportunity; medium values of asset substitution constraint; low values of asset specificity; low values of fundamental risk of loan portfolios; and low values of the liquidity benefit of bank debtholders. In other words, the fair value regime dominates the historical cost regime for extremely high or extremely low values of asset substitution opportunity, extremely high or extremely low values of asset substitution constraint, high values of asset specificity, high values of fundamental risk of loan portfolios; and high values of the liquidity benefit of bank debtholders.

Our investigation contributes to the *public policy* debate on *prudential regulation and bank accounting*. For example, to the extent that the asset substitution opportunity varies with the phases of business cycles, our result argues for cycle-contingent accounting, if practical. Specifically, if the asset substitution opportunity is larger in economic upturns and smaller in downturns, it is optimally to implement fair value accounting in cycle turns (both the peaks and troughs) and historical cost accounting in other phases of business cycles. Another implication is related to stricter leverage requirements in the Dodd-Frank Act and Basel III. Our result suggests that regulatory leverages be contingent on specific values of relevant parameters, and an across-the-board tightening of regulations may not be value-enhancing.

More generally, this study identifies the characteristics of bank assets and liabilities that should be taken into account in bank regulation and accounting standard setting. In addition, it provides a specific example of how prudential regulation and accounting standard setting should be optimally coordinated.

Our study contributes to the theoretical literature on accounting for banks. (1) This study incorporates *both the asset side and the liability side* of the banks' balance sheets in order to shed light on the debate about the bank opacity described earlier. (2) This study focuses on regulatory coordinations: how are *prudential regulation and accounting standard setting* optimally coordinated?

Most of the existing studies do not focus on regulatory coordinations. Allen and Carletti (2008) focus on historical cost versus fair value accounting as well. In contrast to this study, they are interested in contagion from the insurance sector to the banking sector as opposed to banks' asset substitution and quality decisions. Plantin, Sapra, and Shin (2008) also focus on historical cost versus fair value accounting for banks. They are interested in banks' asset sales decision as opposed to banks' asset substitution and quality decisions. Burkhardt and Strausz (2009) focus on historical cost versus impairment accounting on asset substitution. In contrast, this study introduces banks' quality decision as well as asset substitution decision, thereby introducing the tradeoff between the two. Corona, Nan, and Zhang (2017) focus on historical cost versus fair value accounting for assets in place as opposed to loan originations. They are interested in banks' lending decision as opposed to banks' asset substitution and quality decisions, which are the key ingredients of our study. Lu, Sapra, and Subramanian (2018) study a setting in which the bank can misreport its performance, and their definition of historical cost accounting is one in which the regulatory leverages ignore the (potentially biased) reports. In contrast, we use the conventional definition of historical cost and fair value accounting: loan origination costs versus current fair value of loans. Heaton, Lucas, and McDonald (2010) investigate the mark-to-market regime and do not introduce the historical cost regime.

Several papers focus on other attributes of accounting as opposed to historical cost versus fair value accounting. For example, Corona, Nan, and Zhang (2015) focus on accounting quality.

This model makes several empirical predictions on how the five aforementioned parameters affect the banks' real decisions (asset substitution and quality of loan portfolios) and leverages in alternative accounting regimes. For example, an empirical prediction is the "leverage pro-cyclicality," which is defined by Adrian and Shin (2010) as the positive association of bank asset growth and bank leverage growth.

A complete description of empirical predictions are summarized in Propositions 6, 9, 10, 11, and 12 for fair value accounting and in Propositions 7, 9, 10, 11, and 12 for historical cost accounting. Thus, one may test these predictions across jurisdictions in which alternative accounting methods are in place. In addition, one may test these predictions around the time of an accounting regime change.

Section 2 describes the model setup. Section 3 analyzes the fair value regime and Section 4 analyzes the historical cost regime. Section 5 compares the two regimes. The proofs of Propositions and Lemmas are contained in the Appendix. Section 6 discusses potential research extensions and summarizes this study.

2 Model

We study a banking setting in which (i) a bank chooses the quality (q) of its loan portfolios in period 1 and makes an asset substitution decision (a) in period 2; (ii) a bank regulator chooses regulatory target leverages (equivalently, capital requirements) for period 1 (M_0) and period 2 (M_1). We investigate how alternative accounting regimes (fair value accounting versus historical cost accounting) affect these real decisions differently.

2.1 The Bank's Decisions

A bank needs to lend an exogenous amount of I to the borrowers and thus I is the loan origination value. The bank makes two non-verifiable decisions sequentially in period 1 (from date 0 to date 1) and in period 2 (from date 1 to date 2). At date 0, the bank chooses the quality (q) of its loan portfolios, and at date 1, it makes an asset substitution decision (a). These two decisions jointly affect the bank's date 2 cash flow $V = XZ$ from its loan portfolios, where q directly affects X and a directly affects Z.

At date 0, the bank screens the loan applications and constructs its loan portfolios. The bank decides on the level of the quality $q \in \{q_L, q_H\}$ of the loan portfolios, where $0 \leq q_L < q_H$. The cost of screening is cq where $c > 0$. The quality q affects X stochastically: $X = e^{q+\sigma_X \eta}$ where η is a random variable following a standard normal distribution. Thus, a higher quality will increase the expected value of X ($\mathbb{E}[X] = e^{q+\frac{1}{2}\sigma_X^2}$) as well as the volatility of X ($Var[X] = \left(e^{\sigma_X^2} - 1\right) e^{2q+\sigma_X^2}$). This captures the idea that a higher expected return is accompanied by a higher risk.

At date 1, the bank makes an asset substitution decision $a \in \{0, 1\}$, where $a = 1$ indicates asset substitution and $a = 0$ indicates no asset substitution.[2] The asset substitution decision a affects Z stochastically: $Z = e^{a(\sigma_Z \eta - k)}$ where η is a random variable following a standard normal distribution and $k > \frac{1}{2}\sigma_Z^2$. Thus, a higher asset substitution a decreases the expected value of Z ($\mathbb{E}[Z] = e^{\frac{1}{2}a^2\sigma_Z^2 - ak}$) and increases the volatility of Z ($Var[Z] = \left(e^{a^2\sigma_Z^2} - 1\right) e^{a^2\sigma_Z^2 - 2ak}$). This captures the idea that asset substitution is value-destroying.

2.2 Leverages

The bank plays a "maturity transformation" role in that it borrows short-term debts from depositors and invests the fund in long-term loan portfolios. Specifically, it takes two periods for its loan portfolios to generate cash flow $V = XZ$ at date 2. However, the bank borrows short-term debt in each period from depositors.

At date 0, the bank promises a maturity value M_0 to be repaid at date 1 in return for proceeds of D_0 received at date 0. In the same fashion, at date 1, the bank promises a maturity value M_1 to be repaid at date 2 in return for proceeds of D_1 received at date 1.

Bank regulators impose regulatory target leverages (equivalently, capital requirements) on banks. Specifically, at date 0, the regulator sets $\{M_0, M_1\}$ that banks must comply. Thus, the regulatory M_0 controls the bank's leverage in period 1 and the regulatory M_1 controls

[2] To engage in asset substitution, the bank can structure off-balance-sheet derivatives transactions that increases loan portfolio risk, and/or loosen the criteria according to which it monitors existing loans, etc.

the bank's leverage in period 2. Because the bank chooses quality q in period 1 and makes asset substitution in period 2, the bank regulation is dynamically targeted: M_1 regulates the bank's asset substitution decision in period 2, and M_0 regulates the bank's quality decision in period 1 taking into account the bank's asset substitution decision in period 2.

2.3 Accounting

Two alternative accounting regimes exist: fair value accounting; historical cost accounting. The two accounting regimes produce different accounting reports only at the interim date (date 1), because at date 0 the game just starts and at date 2 the game ends and the cash flow is realized, so both regimes will report the realized cash flow from the loan portfolio.

At date 1, the bank privately learns the realized value of X, where X is a component of the date 2 cash flow $V = XZ$. Thus, the realization of X resolves some of the uncertainty about V. In the fair value regime, the bank is required to report the present value of the realized value of X.[3] In contrast, in the historical cost regime, the bank carries its date 0 reported loan origination value I over to date 1, that is, it does not report the realized value of X. Thus, we consider a pure fair value regime and a pure historical cost regime in this paper, as done by Allen and Carletti (2008) and Plantin, Sapra, and Shin (2008), among others.[4]

2.4 Time Line

The sequence of events is as follows:

- Date 0.

- The bank regulator sets regulatory leverage targets: initial leverage M_0; interim leverage M_1.

[3]This is a Level 3 fair value report, according to the U.S. GAAP, because the realized value of X is the bank's private information.

[4]we discuss regimes with mixed attributes (such as the lower of historical cost or fair value) in the Conclusions section.

- The bank issues the period 1 debt with a date 1 maturity value of M_0 in return for proceeds of D_0 at date 0.
- The bank chooses the quality q of its loan portfolios, where cq is the cost of screening the loan applications.
- Date 1:
- The accounting report is released according to the accounting regime in place.
- The bank issues the period 2 debt with a date 2 maturity value of M_1 in return for proceeds of D_1 at date 1.
- The bank makes its asset substitution decision a.
- Date 2:
- The cash flow V from the bank's loan portfolio is realized.

2.5 Payoffs

The investors demand for liquid debt securities such as demand deposits and the associated banking services. It is the banks' expertise to meet such demand and provide liquidity benefits to investors (Dewatripont and Tirole 1995). Thus, investors' payoff from demand deposits consists of two components: a pecuniary benefit; a non-pecuniary benefit due to liquidity benefit. For example, the investors' payoff from the period 2 debt is $(1 + \lambda)M_1$, where M_1 is the pecuniary benefit and λM_1 is the non-pecuniary benefit, where λ represents the liquidity benefits provided by banks to investors. Empirically, λ is measured as the "bank deposit discount," that is, the difference between the risk-free interest rate and the bank's interest rate on demand deposits (Subramanian and Yang (2018) calibrate λ and find that on average it is about 25%).

At date 0, the bank receives the proceeds of the period 1 debt (D_0) from the debtholders and incur the cost of quality (cq).

At date 1, if the bank value at date 1 (the proceeds from the new debt issuance, D_1, plus the date 1 equity market value E_1) exceeds the bank's liability M_0 at date 1, the bank will pay M_0 to the debtholders. Moreover, if the proceeds from the new debt issuance, D_1, exceeds M_0, the bank pays out $D_1 - M_0$ as dividends to shareholders. The debtholders' payoff in this case will be $(1 + \lambda)M_0$ (which consists of both pecuniary and non-pecuniary benefits), and they also transfer an amount of D_1 to the bank.

However, at date 1, if the bank value at date 1 (D_1 plus E_1) falls short of the bank's liability M_0 at date 1, the bank will be bankrupt, and the shareholders of the bank will receive nothing and the debtholders will receive $\alpha_1 X$. Because the date 1 liquidation of bank assets will result in a much lower liquidation value than the date 2 liquidation, α_1 is normalized to 0 without the loss of generality.

At date 2, the cash flow V from the bank's loan portfolio is realized. If the bank value V exceeds the bank's liability M_1, the bank will pay M_1 to the debtholders. The debtholders' utility in this case will be $(1 + \lambda)M_1$ (which consists of both pecuniary and non-pecuniary benefits).

However, if the bank value V at date 2 falls short of the bank's liability M_1 at date 2, the bank will be bankrupt, and the shareholders of the bank will receive nothing and the debtholders will receive $(1 + \lambda)\alpha V$, where $1 - \alpha \in (0, 1)$ indexes the bankruptcy cost, which arises from the specificity of the bank's assets.

2.6 Critical Parameters

Five parameters are critical in this banking setting. Four of them are attributes of bank assets and one is an attribute of bank liabilities.

- Asset substitution opportunity, σ_Z.
- Asset substitution constraint, k.
- Fundamental risk, σ_X.
- Asset specificity, $1 - \alpha$.

- Liquidity benefits to depositors, λ.

3 Fair Value Accounting

Using backward induction, we first solve for the bank's optimal asset substitution decision, and then for the bank's optimal quality decision, and finally, the regulator's optimal leverage decision.

At date 1, given M_1 set by the regulator and the realized value of X, to maximize the date 1 equity value E_1, the bank makes its asset substitution decision $a \in \{0,1\}$, where $a = 1$ represents asset substitution and $a = 0$ represents no asset substitution:

$\underset{a}{max} \;\; E_1$ where $E_1 = \mathbb{E}[\mathbf{1}_{V \geq M_1} \bullet (V - M_1)]$.

The bank will get a nonnegative equity value when the date 2 cash flow $V = XZ$ exceeds the bank's date 2 liability M_1, the maturity value of the period 2 debt.

Proposition 1. *The bank chooses asset substitution ($a = 1$) if $\frac{M_1}{X} > \gamma_0$ and no asset substitution ($a = 0$) if $\frac{M_1}{X} \leq \gamma_0$, where $\gamma_0 \in (0,1)$ is defined by $1-\gamma_0 = \int_{(k+ln\gamma_0)/\sigma_Z}^{\infty} \left(e^{\sigma_Z \eta - k} - \gamma_0\right) \varphi(\eta) d\eta$. $\frac{\partial \gamma_0}{\partial \sigma_Z} < 0$ and $\frac{\partial \gamma_0}{\partial k} > 0$.*

Proposition 1 states that a higher leverage ($\frac{M_1}{X} > \gamma_0$) will boost the bank's incentive to use depositors' money to gamble (asset substitution). Moreover, a higher asset substitution opportunity (a higher value of σ_Z) will increase the upside potential for the bank and thereby boosting the bank's asset substitution incentive. In a similar vein, a lower asset substitution constraint (a lower value of k) will boost the bank's asset substitution incentive.

The date 1 bank value consists of the debt value D_1 and the equity value E_1 where $D_1 = \mathbb{E}[\mathbf{1}_{V \geq M_1} \bullet (1+\lambda)M_1 + \mathbf{1}_{V < M_1} \bullet (1+\lambda)\alpha V]$ and $E_1 = \mathbb{E}[\mathbf{1}_{V \geq M_1} \bullet (V - M_1)]$. The date 1 debt value D_1 is the expected date 2 payoff to debtholders: when the bank is solvent at date 2 (that is, when the date 2 bank cash flow V exceeds the date 2 bank liability M_1), the debtholders will receive $(1+\lambda)M_1$; however, when the bank is bankrupt at date 2, the debtholders will receive $(1+\lambda)\alpha V$. The date 1 equity value E_1 is the expected date 2 payoff to shareholders: when the bank is solvent at date 2, the shareholders will receive $V - M_1$.

Lemma 1. *The date 1 bank value* $D_1 + E_1 = B_a\left(\frac{M_1}{X}\right)X$ *where* $a \in \{0, 1\}$*:*

When $a = 0$,

$$B_0\left(\frac{M_1}{X}\right) \equiv 1 + \lambda\frac{M_1}{X}; \tag{1}$$

and when $a = 1$,

$$\begin{aligned} B_1\left(\tfrac{M_1}{X}\right) &\equiv \int_{\left(k+ln\frac{M_1}{X}\right)/\sigma_Z}^{\infty}\left(e^{\sigma_Z\eta-k} + \lambda\tfrac{M_1}{X}\right)\varphi(\eta)d\eta \\ &+ \int_{-\infty}^{\left(k+ln\frac{M_1}{X}\right)/\sigma_Z}(1+\lambda)\alpha e^{\sigma_Z\eta-k}\varphi(\eta)d\eta. \end{aligned} \tag{2}$$

At date 0, the bank chooses the quality of its loan portfolios to maximize its date 0 equity value E_0:

$\underset{q}{max} \quad E_0 \quad$ where $E_0 = -cq + \mathbb{E}[\mathbf{1}_{D_1+E_1\geq M_0} \bullet (D_1 + E_1 - M_0)]$

The date 0 equity value E_0 is the expected date 1 bank value (D_1+E_1) less the repayment of the period 1 debt (M_0) if the bank is solvent $(D_1 + E_1 \geq M_0)$ at date 1, less the cost of quality (cq).

Proposition 2. *The bank chooses* q_H *if*

$$\begin{aligned} B_a\left[\int_{\left(ln\frac{M_0}{B_a}-q_H\right)/\sigma_X}^{\infty}\left(e^{q_H+\sigma_X\eta} - \tfrac{M_0}{B_a}\right)\varphi(\eta)d\eta - \int_{\left(ln\frac{M_0}{B_a}-q_L\right)/\sigma_X}^{\infty}\left(e^{q_L+\sigma_X\eta} - \tfrac{M_0}{B_a}\right)\varphi(\eta)d\eta\right] \\ \geq c(q_H - q_L) \end{aligned} \tag{3}$$

and chooses q_L *otherwise, where* B_a *is defined in Lemma 1.*

The bank is more likely to choose q_L *when* M_0 *is higher.*

Proposition 2 states that debt overhang (a higher value of M_0) will likely mitigate the bank's incentive for choosing a higher quality of its loan portfolios.

At date 0, the regulator sets the regulatory target leverages for both date 0 and date 1: $\{M_0, M_1\}$. In the fair value regime, the date 1 accounting report discloses the realized value of X, and thus M_1 can be tied to the realized value of X. The regulator set these leverage targets to maximize the date 0 bank value, which consists of the date 0 debt value (D_0) and the date 0 equity value (E_0):

$\max_{M_0, M_1} \; D_0 + E_0$ where $E_0 = -cq + \mathbb{E}[\mathbf{1}_{D_1+E_1 \geq M_0} \bullet (D_1 + E_1 - M_0)] = -cq + \mathbb{E}[\mathbf{1}_{B_a X \geq M_0} \bullet (B_a X - M_0)]$ and $D_0 = \mathbb{E}[\mathbf{1}_{D_1+E_1 \geq M_0} \bullet (1+\lambda)M_0] = \mathbb{E}[\mathbf{1}_{B_a X \geq M_0} \bullet (1+\lambda)M_0]$.

The date 0 equity value E_0 is the expected date 1 bank value ($D_1 + E_1$) less the repayment of the period 1 debt (M_0) if the bank is solvent ($D_1 + E_1 = B_a X \geq M_0$) at date 1, less the cost of quality (cq), and the date 0 debt value D_0 is the maturity value of the period 1 debt (M_0) plus the liquidity benefits associated with it (λM_0) if the bank is solvent ($D_1 + E_1 = B_a X \geq M_0$) at date 1.

Proposition 3. *(Fair Value (FV) Regime) In equilibrium, the regulator's choice of the regulatory target leverages $\{M_0^{FV}, M_1^{FV}\}$, the bank's period 1 quality decision q^{FV} and period 2 asset substitution decision a^{FV} are as follows:*

(i) If $B_1(\gamma_1) > B_0(\gamma_0)$ and $B_1(\gamma_1)\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta \geq c$, $M_0^{FV} = e^{q_H+\sigma_X T}B_1(\gamma_1)$, $M_1^{FV} = \gamma_1 X$, $q^{FV} = q_H$, and $a^{FV} = 1$.

(ii) If $B_1(\gamma_1) > B_0(\gamma_0)$ and $B_1(\gamma_1)\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta < c$, $M_0^{FV} = e^{q_L+\sigma_X T}B_1(\gamma_1)$, $M_1^{FV} = \gamma_1 X$, $q^{FV} = q_L$, and $a^{FV} = 1$.

(iii) If $B_0(\gamma_0) > B_1(\gamma_1)$ and $B_0(\gamma_0)\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta \geq c$, $M_0^{FV} = e^{q_H+\sigma_X T}B_0(\gamma_0)$, $M_1^{FV} = \gamma_0 X$, $q^{FV} = q_H$, and $a^{FV} = 0$.

(iv) If $B_0(\gamma_0) > B_1(\gamma_1)$ and $B_0(\gamma_0)\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta < c$, $M_0^{FV} = e^{q_L+\sigma_X T}B_0(\gamma_0)$, $M_1^{FV} = \gamma_0 X$, $q^{FV} = q_L$, and $a^{FV} = 0$.

In the above,γ_1 is defined by

$$h\left(\left(k + ln\gamma_1\right)/\sigma_Z\right)/\sigma_Z = \frac{\lambda}{(1+\lambda)(1-\alpha)}, \tag{4}$$

in which $h() \equiv \frac{\varphi()}{1-\Phi()}$ is the hazard rate, and T is defined by

$$h(T)/\sigma_X = \frac{\lambda}{1+\lambda}. \tag{5}$$

The date 0 bank value is $D_0^{FV} + E_0^{FV} = -cq^{FV} + B_{a^{FV}}(\gamma_{a^{FV}})e^{q^{FV}}\int_T^\infty \left(e^{\sigma_X \eta} + \lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta$.

Proposition 3 shows that, to maximize the date 0 bank value, the regulator optimally chooses the regulatory target leverages to induce the bank's period 1 quality decision and period 2 asset substitution decision.

(1) Proposition 3 states the conditions of regulatory forbearance in which the bank's asset substitution ($a^{FV} = 1$) and/or low quality choice ($q^{FV} = q_L$) is tolerated by the regulator.

(2) In the fair value regime, because at date 1 the accounting report discloses the realized value of X, the interim target leverage M_1^{FV} can be tied to the fair value disclosure ($M_1^{FV} = \gamma_1 X$ or $M_1^{FV} = \gamma_0 X$).

(3) The regulator sets the initial target leverage in the form of $M_0^{FV} = e^{q+\sigma_X T} B_a(\gamma_a)$ in order to tailor this target to the desired combination of the bank's period 1 quality decision (q) and period 2 asset substitution decision (a).

4 Historical Cost Accounting

In the historical cost accounting regime, the bank's asset substitution decision is the same as that in the fair value regime because the two regimes differ only in what is reported to outsiders at date 1 rather than what is known by the bank at date 1. At date 1, in both regimes, the bank knows the realized value of X and thus its asset substitution decision is described by Proposition 1: the bank chooses $a = 1$ if $X < \frac{M_1}{\gamma_0}$ and $a = 0$ if $X \geq \frac{M_1}{\gamma_0}$.

At date 1, in the historical cost regime, the accounting report does not update the date 0 report, which is the loan origination value. Thus, at date 1, the debtholders and shareholders in the capital market do not know the realized value of X. Thus, the date 1 market value of debt is a weighted average of the debt value given $a = 1$ and the debt value given $a = 0$: $D_1 = \int_0^{\frac{M_1}{\gamma_0}} D_1\left(\frac{M_1}{X}; a = 1\right) f(X)dX + \int_{\frac{M_1}{\gamma_0}}^{\infty} D_1\left(\frac{M_1}{X}; a = 0\right) f(X)dX$. Similarly, the date 1 market value of equity is a weighted average of the equity value given $a = 1$ and the equity value given $a = 0$:$E_1 = \int_0^{\frac{M_1}{\gamma_0}} E_1\left(\frac{M_1}{X}; a = 1\right) f(X)dX + \int_{\frac{M_1}{\gamma_0}}^{\infty} E_1\left(\frac{M_1}{X}; a = 0\right) f(X)dX$. Thus, the date 1 market value of the bank is a weighted average of the bank value given $a = 1$ ($B_1\left(\frac{M_1}{X}\right) X$) and the bank value given $a = 0$ ($B_0\left(\frac{M_1}{X}\right) X$):

$$EBX \equiv D_1 + E_1 = \int_0^{\frac{M_1}{\gamma_0}} B_1\left(\frac{M_1}{X}\right) Xf(X)dX + \int_{\frac{M_1}{\gamma_0}}^{\infty} B_0\left(\frac{M_1}{X}\right) Xf(X)dX. \quad (6)$$

At date 0, the bank chooses q to maximize the date 0 equity value $E_0 = -cq + \mathbb{E}[\mathbf{1}_{D_1+E_1 \geq M_0} \bullet (D_1 + E_1 - M_0)] = -cq + \mathbb{E}[\mathbf{1}_{EBX \geq M_0} \bullet (EBX - M_0)]$.

To preclude the unrealistic case in which the bank always chooses q_L, from now on, we focus on the case in which $EBX(q_H) - EBX(q_L) \geq c(q_H - q_L)$, which states that the marginal benefit from a higher value of q exceeds the marginal cost.

Proposition 4. *The bank chooses q_H if $M_0 \leq EBX(q_H) - c(q_H - q_L)$ and chooses q_L otherwise, where EBX is defined in (6).*

The bank is more likely to choose q_L when M_0 is higher.

Proposition 4 states that debt overhang (a higher value of M_0) will likely mitigate the bank's incentive for choosing a higher quality of its loan portfolios.

At date 0, the regulator sets the regulatory target leverages for both date 0 and date 1: $\{M_0, M_1\}$. In the historical cost regime, the date 1 accounting report does not disclose the realized value of X, and thus M_1 cannot be tied to the realized value of X. The regulator set these leverage targets to maximize the date 0 bank value, which consists of the date 0 debt value (D_0) and the date 0 equity value (E_0):

$\max_{M_0, M_1} \; D_0 + E_0$ where $E_0 = -cq + \mathbb{E}[\mathbf{1}_{D_1+E_1 \geq M_0} \bullet (D_1 + E_1 - M_0)] = -cq + \mathbb{E}[\mathbf{1}_{EBX \geq M_0} \bullet (EBX - M_0)]$ and $D_0 = \mathbb{E}[\mathbf{1}_{EBX \geq M_0} \bullet (1 + \lambda)M_0]$.

The date 0 equity value E_0 is the expected date 1 bank value ($D_1 + E_1 = EBX$) less the repayment of the period 1 debt (M_0) if the bank is solvent ($D_1 + E_1 = EBX \geq M_0$) at date 1, less the cost of quality (cq), and the date 0 debt value D_0 is the maturity value of the period 1 debt (M_0) plus the liquidity benefits associated with it (λM_0) if the bank is solvent ($D_1 + E_1 = EBX \geq M_0$) at date 1.

Proposition 5. *(Historical Cost (HC) Regime) In equilibrium, the regulator's choice of the* *regulatory target leverages* $\{M_0^{HC}, M_1^{HC}\}$*, the bank's period 1 quality decision* q^{HC} *and period 2 asset substitution decision* a^{HC} *are as follows:* $M_0^{HC} = EBX(q_H, M_1^{HC}) - c(q_H - q_L)$*,* M_1^{HC} *is defined by (7),*

$$\int_0^{\frac{M_1^{HC}}{\gamma_0}} B_1'\left(\frac{M_1^{HC}}{X}\right) f(X)dX + \int_{\frac{M_1^{HC}}{\gamma_0}}^{\infty} B_0'\left(\frac{M_1^{HC}}{X}\right) f(X)dX = 0, \tag{7}$$

$q^{HC} = q_H$*,* $a^{HC} = 1$ *if* $X < \frac{M_1^{HC}}{\gamma_0}$ *or* $a^{HC} = 0$ *if* $X \geq \frac{M_1^{HC}}{\gamma_0}$*. The date 0 bank value is* $D_0^{HC} + E_0^{HC} = -cq_H + (1+\lambda)EBX(q_H, M_1^{HC}) - \lambda c(q_H - q_L)$*. In the above,* $EBX(q_H, M_1^{HC})$ *is the value of* EBX *in (6) evaluated at* $q = q_H$ *and* $M_1 = M_1^{HC}$*.*

Proposition 5 shows that, to maximize the date 0 bank value, the regulator optimally chooses the regulatory target leverages to induce the bank's period 1 quality decision and period 2 asset substitution decision. Several observations are in order.

(1) In the historical cost regime, because the accounting report at date 1 does not disclose the realized value of X, the interim target leverage M_1^{FV} cannot be tied to a specific value of X, as demonstrated by (7). That is why, in setting M_1, the regulator must take into account the probability that the realized value of X is low (and thus may triggering the bank's asset substitution) and high (and thus may not trigger it). In contrast, in the fair value regime, the regulator can tie M_1 to a specific value of X.

(2) The regulator may tolerate the bank's asset substitution ($a^{HC} = 1$). However, this regulatory forbearance is not deterministic but random because the regulator cannot tie M_1 to the realized value of X. In contrast, in the fair value regime, this regulatory forbearance is deterministic because the regulator can tie M_1 to the realized value of X.

(3) The regulator does not tolerate the low quality choice ($q^{HC} = q_L$) but always induce the high quality q_H. Because the regulator cannot induce her desired asset substitution decision of the bank at will, she chooses to at least make sure that the high rather than low quality be chosen by the bank. In contrast, in the fair value regime, the regulator may find it optimal to induce either the high or the low quality.

(4) To avoid imposing too much debt overhang on the bank, the regulator will not set a high initial leverage target M_0 . To the extent that the high quality will be induced, the regulator will set the initial target leverage as high as possible in order to increase the scale (λM_0) of the debtholders' liquidity benefit.

In contrast, in the fair value regime, the regulator does not tailor this target to the desired combination of the bank's period 1 quality decision (q) and period 2 asset substitution decision (a), for the reasons given in (2) and (3) above.

5 Fair Value Accounting versus Historical Cost Accounting

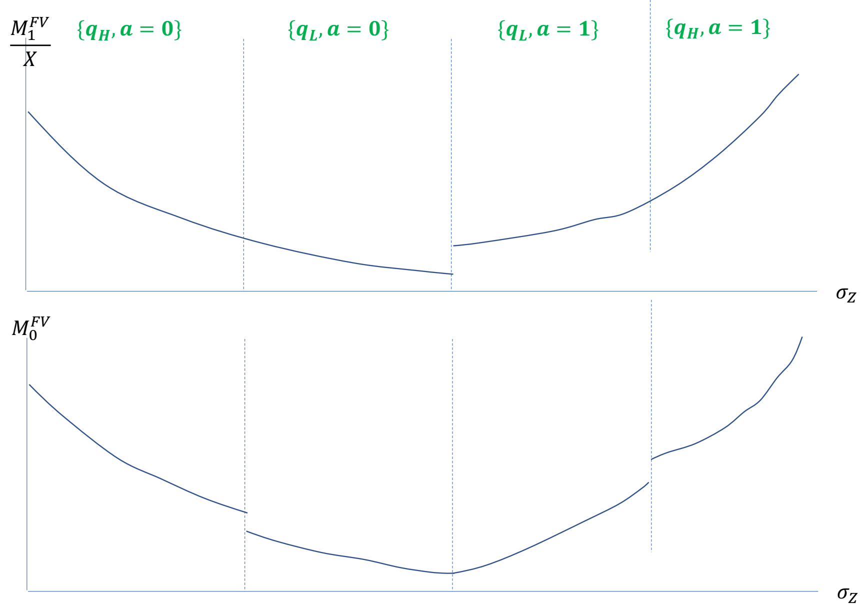
we compare the fair value accounting and the historical cost accounting in terms of the following five parameters. Four of them are attributes of bank assets and one is an attribute of bank liabilities.

The regulator takes into account three economic factors in setting the bank's leverage targets: the bank's asset substitution decision in period 2; the bank's quality decision in period 1; and the debtholders' liquidity benefits from deposits. The regulator trades off these economic forces in order to maximize the date 0 bank value.

5.1 Asset Substitution Opportunity

Figure 1 and Proposition 6 describe the effects of asset substitution opportunity σ_Z.

Figure 1. The Effects of Asset Substitution Opportunity σ_Z.



Proposition 6. *In the fair value regime (FV), in terms of asset substitution opportunity σ_Z, the equilibrium values of M_0^{FV}, $\frac{M_1^{FV}}{X}$, q^{FV}, and a^{FV} are as follows, where the cutoff values of σ_Z are defined in the proof:*

	$\sigma_Z \leq \sigma_Z^I$	$\sigma_Z \in (\sigma_Z^I, \sigma_Z^{II}]$
M_0^{FV}	$e^{q_H+\sigma_X T} B_0(\gamma_0)$ [$\downarrow$ *in* σ_Z]	$e^{q_L+\sigma_X T} B_0(\gamma_0)$ [$\downarrow$ *in* σ_Z]
q^{FV}	q_H	q_L
M_1^{FV}	$\gamma_0 X$ [$\downarrow$ *in* σ_Z]	$\gamma_0 X$ [$\downarrow$ *in* σ_Z]
a^{FV}	0	0

	$\sigma_Z \in (\sigma_Z^{II}, \sigma_Z^{III})$	$\sigma_Z \geq \sigma_Z^{III}$
M_0^{FV}	$e^{q_L+\sigma_X T} B_1(\gamma_1)$ [$\uparrow$ *in* σ_Z]	$e^{q_H+\sigma_X T} B_1(\gamma_1)$ [$\uparrow$ *in* σ_Z]
q^{FV}	q_L	q_H
M_1^{FV}	$\gamma_1 X$ [$\uparrow$ *in* σ_Z]	$\gamma_1 X$ [$\uparrow$ *in* σ_Z]
a^{FV}	1	1

For low values of σ_Z ($\sigma_Z \leq \sigma_Z^I$), the asset substitution opportunity is low and therefore the bank's asset substitution incentive is low. When σ_Z is higher, asset substitution becomes more attractive to the bank, and therefore the regulator decreases the interim leverage ($\frac{M_1^{FV}}{X} = \gamma_0$) to rein in asset substitution so that $a^{FV} = 0$. A low asset substitution mitigates the bank's incentive to invest in quality. Thus, the regulator decreases the initial leverage M_0^{FV} to reduce debt overhang in order to encourage the bank's quality investment ($q^{FV} = q_H$). However, when σ_Z becomes even higher ($\sigma_Z \in (\sigma_Z^I, \sigma_Z^{II}]$), the bank's opportunity cost of no asset substitution in period 2 becomes even higher. Anticipating this, the bank's period 1 incentive in quality becomes reduced ($q^{FV} = q_L$).

For high values of σ_Z ($\sigma_Z \in (\sigma_Z^{II}, \sigma_Z^{III})$), if the regulator wishes to curb asset substitution, she needs to further decrease the interim leverage. However, this would further decrease the depositors' liquidity benefits. Thus, the regulator forebears asset substitution ($a^{FV} = 1$) by increasing the interim leverage ($\frac{M_1^{FV}}{X} = \gamma_1$). The bank's asset substitution incentive keeps increasing when σ_Z becomes even higher ($\sigma_Z \geq \sigma_Z^{III}$). A high asset substitution stimulates the bank's incentive to invest in quality ($q^{FV} = q_H$). Because debt overhang is less a concern, the regulator increases the initial leverage M_0^{FV} to increase the depositors' liquidity benefits.

Proposition 6 implies the following in the fair value regime:

- The optimal regulatory leverages (M_0^{FV} and $\frac{M_1^{FV}}{X}$) are U-shaped in asset substitution opportunity σ_Z (Figure 1).

- In particular, for high values of asset substitution opportunity σ_Z ($\sigma_Z \geq \sigma_Z^{II}$), the higher the value of σ_Z, the higher the optimal regulatory leverages. This result is opposite to the conventional wisdom, which advocates a lower leverage for a higher value of asset substitution opportunity.

- To the extent that the asset substitution opportunity σ_Z varies with the phases of business cycles, Proposition 6 argues for cycle-contingent regulatory leverages. Specifically, if the asset substitution opportunity σ_Z is larger in economic upturns and smaller in downturns, Proposition 6 argues for a higher regulatory leverage in cycle turns (both the peaks and troughs) and a lower leverage in other phases of business cycles.

Proposition 7. *In the historical cost regime (HC), M_0^{HC}, M_1^{HC}, and $Pr(a^{HC} = 1)$ are increasing in asset substitution opportunity σ_Z, while $q^{HC} = q_H$.*

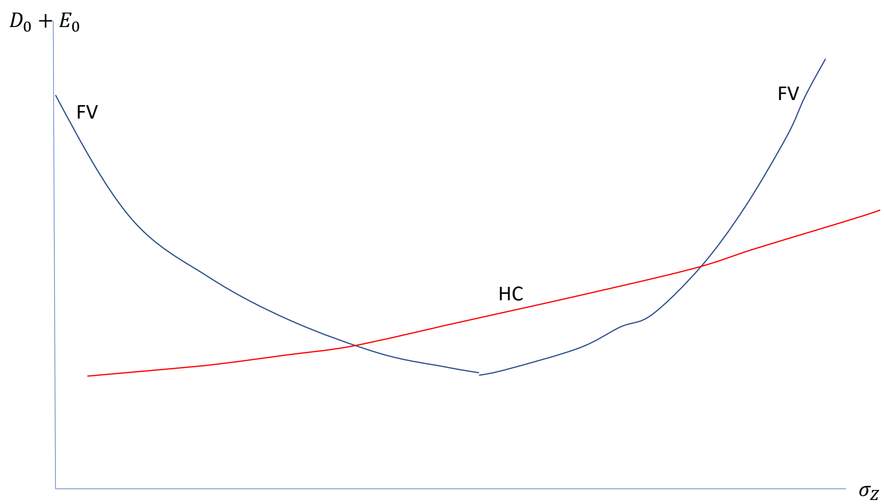
A larger asset substitution opportunity σ_Z will boost the bank's asset substitution incentive ($Pr(a^{HC} = 1)$). Because in the historical cost regime, the regulator always wants to induce the bank to choose the high quality ($q^{HC} = q_H$ in Proposition 5), she tolerates asset substitution by increasing M_1^{HC} when the bank's asset substitution incentive increases. The larger interim leverage has an additional benefit of increasing the depositors' liquidity benefits. Because debt overhang is less a concern, the regulator increases the initial leverage M_0^{HC} to increase the depositors' liquidity benefits.

Proposition 7 implies the following in the historical cost regime:

- The optimal regulatory leverages (M_0^{HC} and M_1^{HC}) are increasing in asset substitution opportunity σ_Z. This result is opposite to the conventional wisdom, which advocates a lower leverage for a higher value of asset substitution opportunity.

- To the extent that the asset substitution opportunity σ_Z varies with the phases of business cycles, Proposition 7 argues for cycle-contingent regulatory leverages. Specifically, if the asset substitution opportunity σ_Z is larger in economic upturns and smaller in downturns, Proposition 7 argues for a higher regulatory leverage in upturns and a lower leverage in downturns. In other words, Proposition 7 argues for "leverage procyclicality," which is defined by Adrian and Shin (2010) as the positive association of bank asset growth and bank leverage growth.

Figure 2. HC versus FV: Asset Substitution Opportunity σ_Z.



Proposition 8. *(HC versus FV) In terms of the date 0 bank value D_0+E_0, the historical cost regime dominates the fair value regime for medium values of asset substitution opportunity σ_Z, whereas the fair value regime dominates the historical cost regime for extremely high or extremely low values of σ_Z.*

Proposition 8 identifies the condition under which the fair value regime dominates the historical cost regime and the condition under which the opposite is true.

For sufficiently low values of asset substitution opportunity σ_Z, although both regimes induce the high quality of the loan portfolios, the fair value regime dominates the historical cost regime for the following reasons implied by Propositions 6 and 7. (1) No asset substitution exists in the fair value regime whereas asset substitution may arise in the historical cost regime. (2) Because the regulator in the historical cost regime has to take into account both asset substitution and no asset substitution, the regulatory leverages and thus the liquidity benefits to the debtholders are higher in the fair value regime than in the historical cost regime.

However, for medium values of asset substitution opportunity σ_Z, although both regimes may induce asset substitution or no asset substitution, the historical cost regime dominates the fair value regime for the following reasons implied by Propositions 6 and 7. (1) The high quality is induced in the historical cost regime whereas the low quality is induced in the fair value regime. (2) Because the regulator in the historical cost regime has to take into account both asset substitution and no asset substitution, the regulatory leverages and thus the liquidity benefits to the debtholders are lower in the fair value regime than in the historical cost regime.

For sufficiently high values of asset substitution opportunity σ_Z, although both regimes induce the high quality of the loan portfolios and both regimes induce similar incidence of asset substitution, the fair value regime dominates the historical cost regime because the regulatory leverages are higher in the fair value regime than in the historical cost regime for the following reasons implied by Propositions 6 and 7. (1) To induce asset substitution in the fair value regime, the regulator must tolerate a high level of the interim leverage M_1. On the other hand, in the historical cost regime, the regulator balances the probability of asset substitution and the probability of no asset substitution and thus it sets a relatively lower interim leverage than that in the fair value regime. (2) Recall from Proposition 5 that, to induce the high quality of loan portfolios in the historical cost regime, the regulator has to restrain the extent of debt overhang, which in turn constrains the regulatory initial leverage M_0. On the other hand, in the fair value regime, because the bank has enough incentive to choose the high quality in the first place due to sufficiently high values of asset substitution opportunity σ_Z, debt overhang is not quite a big concern and thus the regulator can afford to choose a high initial leverage.

Proposition 8 has the following implications:

- No one accounting regime unconditionally dominates the other in terms of asset substitution opportunity σ_Z (Figure 2).
- In particular, for medium values of asset substitution opportunity σ_Z, the historical cost regime is optimal. In other words, withholding timely information (the realized value of X) is optimal for medium values of σ_Z. This result is inconsistent with the conventional wisdom, which advocates fair value accounting for the purpose of timely recognizing potential bank problems.

- To the extent that the asset substitution opportunity σ_Z varies with the phases of business cycles, Proposition 8 argues for cycle-contingent accounting. Specifically, if the asset substitution opportunity σ_Z is larger in economic upturns and smaller in downturns, Proposition 9 argues for fair value accounting in cycle turns (both the peaks and troughs) and historical cost accounting in other phases of business cycles.

Proposition 9 describe the effects of asset substitution constraint k.

Proposition 9. *(HC versus FV) In terms of the date 0 bank value $D_0 + E_0$, the historical cost regime dominates the fair value regime for medium values of asset substitution constraint k, whereas the fair value regime dominates the historical cost regime for extremely high or extremely low values of k.*

Because the asset substitution constraint k works against the asset substitution opportunity σ_Z, the intuition and the implications for Proposition 9 is the same as that for Proposition 8.

5.2 Asset Specificity

Proposition 10 describe the effects of asset specificity $1 - \alpha$, where $(1 - \alpha)V$ is the value lost upon bankruptcy at date 2. Because asset specificity hampers a borrower's ability to borrow debt in favorable terms, $1 - \alpha$ can be also interpreted as debt capacity.

Proposition 10. *(HC versus FV) In terms of the date 0 bank value $D_0 + E_0$, the historical cost regime dominates the fair value regime for high values of asset specificity $1 - \alpha$, whereas the fair value regime dominates the historical cost regime for low values of $1 - \alpha$.*

In both regimes, when bank assets are very specific, the bank has a small debt capacity, and therefore the regulatory leverages are low. The low interim leverage induces a low date 1 debt value, which dampens the bank's date 0 incentive for quality. In addition, a low interim leverage induces a low incidence of asset substitution.

In the fair value regime, because the realized value of X is reported at date 1, the regulator can directly tie its interim target leverage to the realized value of X and thus she can induce her desired asset substitution decision for certain. However, in the historical cost regime, because the realized value of X is not reported, the regulator cannot directly tie its interim target leverage to the realized value of X and thus she cannot induce her desired asset substitution decision for certain. Thus, she has to take into account both the probability of asset substitution and the probability of no asset substitution.

When bank assets are very specific, the historical cost regime dominates the fair value regime for the following reasons.

In the fair value regime, the regulator sets a low interim leverage to rule out asset substitution. The dampened asset substitution incentive in period 2 mitigates the bank's period 1 incentive for high quality. To motivate the high quality, the regulator has to set a low initial leverage.

In the historical cost regime, the regulator sets an interim leverage which is relatively higher than that in the fair value regime. This helps the regulator to induce the high quality. Thus, the debt overhang is not quite a concern and she can afford to set a relatively high initial leverage in order to enhance the debtholders' liquidity benefit.

In brief, when bank assets are very specific, in the historical cost regime, both the quality and the leverages are higher than their counterparts in the fair value regime, and therefore the date 0 bank value is higher in the historical cost regime. The converse is true when bank assets are not very specific.

5.3 Fundamental Risk

Proposition 11. *(HC versus FV) In terms of the date 0 bank value $D_0 + E_0$, the historical cost regime dominates the fair value regime for low values of fundamental risk σ_X, whereas the fair value regime dominates the historical cost regime for high values of σ_X. In particular,*

(i) in the fair value regime, for $\sigma_X \geq max\{h(T^*), \sqrt{\frac{2}{\pi}}\}/\left(\frac{\lambda}{1+\lambda}\right)$ where $h'(T^*) = \frac{\lambda}{1+\lambda}$, the higher the value of σ_X, the higher the M_0^{FV} and the more likely the bank chooses q_H; and

(ii) in the historical cost regime, M_0^{HC}, M_1^{HC}, $Pr(a^{HC} = 1)$ are decreasing in σ_X.

In the fair value regime, the higher the fundamental risk, the larger the upside potential, and the larger the bank's incentive to invest in quality. Because debt overhang is less a concern, the social planner increases the initial leverage to increase the depositors' liquidity benefits.

In the historical cost regime, however, a higher fundamental risk will magnify asset substitution incentive induced by σ_Z and therefore, the regulator will decrease the interim leverage in order to decrease the incidence of asset substitution. A dampened asset substitution incentive in period 2 will dampen the bank's incentive for a high quality choice in period 1. To encourage the bank to choose the high quality, the regulator reduces the initial leverage to reduce debt overhang.

The above comparison of the two regimes implies that for high values of σ_X, even though both regimes induce the high quality, the regulatory leverages in the fair value regime is relatively higher than those in the historical cost regime and thus the fair value regime dominates. The converse is true for low values of fundamental risk σ_X.

5.4 Liquidity Benefit

The depositors' liquidity benefit ("convenience spread") is represented by λ. The following proposition identifies the condition under which the historical cost regime dominates the fair value regime in terms of λ and the condition under which the converse is true.

Proposition 12. *(HC versus FV) In terms of the date 0 bank value $D_0 + E_0$, the historical cost regime dominates the fair value regime for low values of debtholders' liquidity benefit λ, whereas the fair value regime dominates the historical cost regime for high values of λ. In particular,*

(i) in the fair value regime,

	$\lambda \leq \lambda^I$	$\lambda \in (\lambda^I, \lambda^{II}]$
M_0^{FV}	$e^{q_H+\sigma_X T} B_0(\gamma_0)$ [$\uparrow$ *in* λ]	$e^{q_L+\sigma_X T} B_0(\gamma_0)$ [$\uparrow$ *in* λ]
q^{FV}	q_H	q_L
M_1^{FV}	$\gamma_0 X$	$\gamma_0 X$
a^{FV}	0	0

	$\lambda \in (\lambda^{II}, \lambda^{III})$	$\lambda \geq \lambda^{III}$
M_0^{FV}	$e^{q_L+\sigma_X T} B_1(\gamma_1)$ *[↑ in λ]*	$e^{q_H+\sigma_X T} B_1(\gamma_1)$ *[↑ in λ]*
q^{FV}	q_L	q_H
M_1^{FV}	$\gamma_1 X$ *[↑ in λ]*	$\gamma_1 X$ *[↑ in λ]*
a^{FV}	1	1

(ii) in the historical cost regime, M_0^{HC}, M_1^{HC}, $Pr(a^{HC} = 1)$ are increasing in λ.

When the depositors' liquidity benefit (λ) is more valuable, the regulator increases the regulatory leverages to increase this social benefit of debt. A higher interim leverage implies that the regulator forebears asset substitution. In addition, a higher initial leverage increases debt overhang and thus mitigates the bank's incentive for quality. However, a heightened incentive of asset substitution stimulates the bank's incentive to invest in quality and thus debt overhang becomes less a concern, therefore the regulator can afford to increase the initial leverage to increase the depositors' liquidity benefit.

In the historical cost regime, the regulator sets the interim leverage by balancing the probability of asset substitution and the probability of no asset substitution. In contrast, in the fair value regime, the regulator sets a low interim leverage for low values of λ and a high interim leverage for high values of λ. Therefore, for low values of λ, the initial leverage (and thus the debtholders' liquidity benefit) is relatively higher in the historical cost regime than in the fair value regime. The converse is true for high values of λ.

6 Conclusions

This study focuses on the bank's decisions (asset substitution and quality of loan portfolios) and on the regulatory coordination (prudential regulation and bank accounting). It identifies several key parameters related to bank assets and bank liabilities and investigates their effects on bank decisions and regulatory decisions, and ultimately, on bank value.

We contrast a pure fair value regime and a pure historical cost regime in the sense that no frictions in accounting recognition and measurement is introduced. In practice, however, a mixed-attributes accounting regime is used. For example, impairment accounting mixes both the feature of a pure historical cost regime and the feature of a pure fair value regime. Therefore, the two pure regimes can serve as a benchmark and a starting point for a future extension of the baseline model in this study to incorporate mixed-attributes regimes.

Appendix: Proofs

PROOF OF PROPOSITION 1

The bank's optimization program is $\underset{a}{max} \quad E_1$ where $E_1 = \mathbb{E}[\mathbf{1}_{V \geq M_1} \bullet (V - M_1)]$. The bank chooses between $a = 1$ and $a = 0$.

(i) The case of $a = 1$.

The condition $V = XZ \geq M_1$ is equivalent to $Z = e^{\sigma_Z \eta - k} \geq \frac{M_1}{X} \Leftrightarrow \eta \geq \left(k + ln\frac{M_1}{X}\right)/\sigma_Z$. Thus, $\frac{E_1}{X} = \int_{\left(k+ln\frac{M_1}{X}\right)/\sigma_Z}^{\infty} \left(e^{\sigma_Z \eta - k} - \frac{M_1}{X}\right)\varphi(\eta)d\eta$, which implies that

$$\frac{\partial \frac{E_1}{X}}{\partial \frac{M1}{X}} = -\left[1 - \Phi\left(\left(k + ln\frac{M_1}{X}\right)/\sigma_Z\right)\right] < 0. \tag{8}$$

At $\frac{M_1}{X} = 0$, $\frac{E_1}{X} = e^{\frac{1}{2}\sigma_Z^2 - k} < 1$, and when $\frac{M_1}{X} \to \infty$, $\frac{E_1}{X} \to 0$.

(ii) The case of $a = 0$.

The condition $V = XZ \geq M_1$ is equivalent to $1 \geq \frac{M_1}{X}$ where $Z = 1$ when $a = 0$. Therefore, if $\frac{M_1}{X} > 1$, $\frac{E_1}{X} = 0$, and if $\frac{M_1}{X} \leq 1$, $\frac{E_1}{X} = 1 - \frac{M_1}{X}$.

Thus, $\frac{E_1}{X}$ given $a = 1$ and $\frac{E_1}{X}$ given $a = 0$ intersect at $\frac{M_1}{X} = \gamma_0 \in (0, 1)$ where γ_0 is defined by

$$1 - \gamma_0 = \int_{(k+ln\gamma_0)/\sigma_Z}^{\infty} \left(e^{\sigma_Z \eta - k} - \gamma_0\right)\varphi(\eta)d\eta. \tag{9}$$

■

PROOF OF LEMMA 1

Recall from the text that

$$D_1 = \mathbb{E}[\mathbf{1}_{V \geq M_1} \bullet (1 + \lambda)M_1 + \mathbf{1}_{V < M_1} \bullet (1 + \lambda)\alpha V]. \tag{10}$$

(i) The case of $a = 1$.

By (10), $\frac{D_1}{X} = \int_{\left(k+ln\frac{M_1}{X}\right)/\sigma_Z}^{\infty}(1 + \lambda)\frac{M_1}{X}\varphi(\eta)d\eta + \int_{-\infty}^{\left(k+ln\frac{M_1}{X}\right)/\sigma_Z}(1 + \lambda)\alpha e^{\sigma_Z \eta - k}\varphi(\eta)d\eta$. The proof of Proposition 1 shows that $\frac{E_1}{X} = \int_{\left(k+ln\frac{M_1}{X}\right)/\sigma_Z}^{\infty} \left(e^{\sigma_Z \eta - k} - \frac{M_1}{X}\right) \varphi(\eta) d\eta$. Therefore, $\frac{D_1}{X} + \frac{E_1}{X} = B_1(\frac{M_1}{X})$.

(ii) The case of $a = 0$.

By (10), $\frac{D_1}{X} = (1+\lambda)\frac{M_1}{X}$. The proof of Proposition 1 shows that $\frac{E_1}{X} = 1 - \frac{M_1}{X}$ because $\frac{M_1}{X} \leq \gamma_0 < 1$. Therefore, $\frac{D_1}{X} + \frac{E_1}{X} = B_0(\frac{M_1}{X})$. ■

PROOF OF PROPOSITION 2

The bank's optimization program is $\underset{q}{max} \quad E_0$ where $E_0 = -cq + \mathbb{E}[\mathbf{1}_{D_1+E_1 \geq M_0} \bullet (D_1 + E_1 - M_0)]$. The condition $D_1 + E_1 \geq M_0$ is equivalent to $B_a X \geq M_0 \Leftrightarrow X = e^{q+\sigma_X \eta} \geq \frac{M_0}{B_a} \Leftrightarrow \eta \geq \left(ln\frac{M_0}{B_a} - q\right)/\sigma_X$. Therefore, E_0 as a function of q can be rewritten as $E_0(q) = -cq + B_a \int_{\left(ln\frac{M_0}{B_a}-q\right)/\sigma_X}^{\infty} \left(e^{q+\sigma_X \eta} - \frac{M_0}{B_a}\right) \varphi(\eta) d\eta$.

The bank chooses q_H if and only if $E_0(q_H) \geq E_0(q_L)$, which is equivalent to (3).

The derivative of the left-hand side of (3) with respect to $\frac{M_0}{B_a}$ is $\Phi\left(\left(ln\frac{M_0}{B_a} - q_H\right)/\sigma_X\right) - \Phi\left(\left(ln\frac{M_0}{B_a} - q_L\right)/\sigma_X\right) < 0$. Therefore, the bank is more likely to choose q_L when M_0 is higher. ■

PROOF OF PROPOSITION 3

At date 0, the regulator's optimization program is $\underset{M_0, M_1}{max} \quad D_0 + E_0$ where $E_0 = -cq + \mathbb{E}[\mathbf{1}_{D_1+E_1 \geq M_0} \bullet (D_1 + E_1 - M_0)]$ and $D_0 = \mathbb{E}[\mathbf{1}_{D_1+E_1 \geq M_0} \bullet (1+\lambda) M_0]$.

The condition $D_1 + E_1 \geq M_0$ is equivalent to $\Leftrightarrow B_a X \geq M_0 \Leftrightarrow X = e^{q+\sigma_X \eta} \geq \frac{M_0}{B_a} \Leftrightarrow \eta \geq \left(ln\frac{M_0}{B_a} - q\right)/\sigma_X$. Therefore, $D_0 = B_a \int_{\left(ln\frac{M_0}{B_a}-q\right)/\sigma_X}^{\infty} (1+\lambda)\frac{M_0}{B_a}\varphi(\eta) d\eta$. From the proof of Proposition 2, $E_0 = -cq + B_a \int_{\left(ln\frac{M_0}{B_a}-q\right)/\sigma_X}^{\infty} \left(e^{q+\sigma_X \eta} - \frac{M_0}{B_a}\right) \varphi(\eta) d\eta$. Therefore,

$$D_0 + E_0 = -cq + B_a \int_{\left(ln\frac{M_0}{B_a}-q\right)/\sigma_X}^{\infty} \left(e^{q+\sigma_X \eta} + \lambda \frac{M_0}{B_a}\right) \varphi(\eta) d\eta. \tag{11}$$

For any given $\{q, a\}$ combination,

$$\frac{\partial(D_0 + E_0)}{\partial \frac{M_1}{X}} = \frac{\partial B_a}{\partial \frac{M_1}{X}} \left[\int_{\left(ln\frac{M_0}{B_a}-q\right)/\sigma_X}^{\infty} e^{q+\sigma_X \eta} \varphi(\eta) d\eta + \frac{M_0}{B_a}(1+\lambda)\varphi\left(\left(ln\frac{M_0}{B_a} - q\right)/\sigma_X\right)/\sigma_X \right].$$

(12)

Because the bracketed term is positive, the sign of $\frac{\partial(D_0+E_0)}{\partial\frac{M_1}{X}}$ is the same as the sign of $\frac{\partial B_a}{\partial\frac{M_1}{X}}$.

(i) For $\frac{M_1}{X} > \gamma_0$, $a = 1$ by Proposition 1 and thus $B_a\left(\frac{M_1}{X}\right) = B_1\left(\frac{M_1}{X}\right)$. $\frac{\partial B_1}{\partial\frac{M_1}{X}} = \left[1-\Phi\left(\left(k+ln\frac{M_1}{X}\right)/\sigma_Z\right)\right]\left[\lambda-(1+\lambda)(1-\alpha)h\left(\left(k+ln\frac{M_1}{X}\right)/\sigma_Z\right)/\sigma_Z\right]$, which is decreasing in $\frac{M_1}{X}$. Therefore, $B_1\left(\frac{M_1}{X}\right)$ achieves its maximum value $B_1(\gamma_1)$ at $\frac{M_1}{X} = \gamma_1$ where γ_1 is defined by (4).

(ii) For $\frac{M_1}{X} \leq \gamma_0$, $a = 0$ by Proposition 1 and thus $B_a\left(\frac{M_1}{X}\right) = B_0\left(\frac{M_1}{X}\right)$. $\frac{\partial B_0}{\partial\frac{M_1}{X}} = \lambda > 0$. Therefore, $B_0\left(\frac{M_1}{X}\right)$ achieves its maximum value $B_0(\gamma_0)$ at $\frac{M_1}{X} = \gamma_0$.

In addition,

$$\frac{\partial(D_0+E_0)}{\partial M_0} = \left[1-\Phi\left(\left(ln\frac{M_0}{B_a(\gamma_a)}-q\right)/\sigma_X\right)\right]\left[\lambda-(1+\lambda)h\left(\left(ln\frac{M_0}{B_a(\gamma_a)}-q\right)/\sigma_X\right)/\sigma_X\right], \tag{13}$$

which is decreasing in M_0. Therefore,

$$M_0(q,a) = e^{q+\sigma_X T}B_a(\gamma_a) \tag{14}$$

where T is defined by (5).

Substituting (14) into (11) yields

$$D_0+E_0 = -cq + B_a(\gamma_a)e^q\int_T^\infty\left(e^{\sigma_X\eta}+\lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta. \tag{15}$$

Because $q \in \{q_L, q_H\}$ and $a \in \{0,1\}$, the regulator chooses one of the four possible cases in order to maximize D_0+E_0:

Case of $\{q_H, a=0\}$: $D_0+E_0 = -cq_H + B_0(\gamma_0)e^{q_H}\int_T^\infty\left(e^{\sigma_X\eta}+\lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta$.

Case of $\{q_H, a=1\}$: $D_0+E_0 = -cq_H + B_1(\gamma_1)e^{q_H}\int_T^\infty\left(e^{\sigma_X\eta}+\lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta$.

Case of $\{q_L, a=0\}$: $D_0+E_0 = -cq_L + B_0(\gamma_0)e^{q_L}\int_T^\infty\left(e^{\sigma_X\eta}+\lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta$.

Case of $\{q_L, a=1\}$: $D_0+E_0 = -cq_L + B_1(\gamma_1)e^{q_L}\int_T^\infty\left(e^{\sigma_X\eta}+\lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta$.

Substituting (14) into (3) in Proposition 2 yields the result that the bank chooses q_H if and only if

$$B_a(\gamma_a)(e^{q_H} - e^{q_L}) \int_T^{\infty} e^{\sigma_X \eta} \varphi(\eta) d\eta \geq c(q_H - q_L). \tag{16}$$

Define $G \equiv \int_T^{\infty} e^{\sigma_X \eta} \varphi(\eta) d\eta$. The bank chooses q in the following way:

The case of $B_1(\gamma_1) > B_0(\gamma_0)$:

If $c(q_H - q_L) \leq B_0(\gamma_0)(e^{q_H} - e^{q_L})G$, according to (16), the bank chooses q_H regardless of a. Because $B_1(\gamma_1) > B_0(\gamma_0)$, the regulator chooses $\frac{M_1}{X} = \gamma_1$ to induce $a = 1$.

If $c(q_H - q_L) > B_1(\gamma_1)(e^{q_H} - e^{q_L})G$, the bank chooses q_L regardless of a. Because $B_1(\gamma_1) > B_0(\gamma_0)$, the regulator chooses $\frac{M_1}{X} = \gamma_1$ to induce $a = 1$.

If $c(q_H - q_L) \in (B_0(\gamma_0)(e^{q_H} - e^{q_L})G, B_1(\gamma_1)(e^{q_H} - e^{q_L})G]$, the bank chooses q_H if $a = 1$ and chooses q_L if $a = 0$. The regulator chooses $\frac{M_1}{X} = \gamma_1$ to induce $a = 1$ and (thus q_H). To see this, note that $D_0 + E_0$ given $\{q_H, a = 1\}$ exceeds $D_0 + E_0$ given $\{q_L, a = 0\}$: $-cq_H + B_1(\gamma_1)e^{q_H} \int_T^{\infty} \left(e^{\sigma_X \eta} + \lambda e^{\sigma_X T}\right) \varphi(\eta) d\eta > -cq_L + B_0(\gamma_0)e^{q_L} \int_T^{\infty} \left(e^{\sigma_X \eta} + \lambda e^{\sigma_X T}\right) \varphi(\eta) d\eta$ $\Leftrightarrow [B_1(\gamma_1)e^{q_H} - B_0(\gamma_0)e^{q_L}] \int_T^{\infty} \left(e^{\sigma_X \eta} + \lambda e^{\sigma_X T}\right) \varphi(\eta) d\eta > c(q_H - q_L)$. The left-hand side of the preceding inequality exceeds $B_1(\gamma_1)(e^{q_H} - e^{q_L}) \int_T^{\infty} \left(e^{\sigma_X \eta} + \lambda e^{\sigma_X T}\right) \varphi(\eta) d\eta$, which is in turn larger than $B_1(\gamma_1)(e^{q_H} - e^{q_L})G$, which is larger than $c(q_H - q_L)$.

In summary, if $B_1(\gamma_1) > B_0(\gamma_0)$, the regulator chooses M_0 and $\frac{M_1}{X}$ to induce $\{q_H, a = 1\}$ if $B_1(\gamma_1)(e^{q_H} - e^{q_L}) \int_T^{\infty} e^{\sigma_X \eta} \varphi(\eta) d\eta \geq c(q_H - q_L)$ and chooses M_0 and $\frac{M_1}{X}$ to induce $\{q_L, a = 1\}$ otherwise.

The case of $B_0(\gamma_0) \geq B_1(\gamma_1)$:

If $c(q_H - q_L) \leq B_1(\gamma_1)(e^{q_H} - e^{q_L})G$, according to (16), the bank chooses q_H regardless of a. Because $B_0(\gamma_0) \geq B_1(\gamma_1)$, the regulator chooses $\frac{M_1}{X} = \gamma_0$ to induce $a = 0$.

If $c(q_H - q_L) > B_0(\gamma_0)(e^{q_H} - e^{q_L})G$, the bank chooses q_L regardless of a. Because $B_0(\gamma_0) \geq B_1(\gamma_1)$, the regulator chooses $\frac{M_1}{X} = \gamma_0$ to induce $a = 0$.

If $c(q_H - q_L) \in (B_1(\gamma_1)(e^{q_H} - e^{q_L})G, B_0(\gamma_0)(e^{q_H} - e^{q_L})G]$, the bank chooses q_L if $a = 1$ and chooses q_H if $a = 0$. The regulator chooses $\frac{M_1}{X} = \gamma_0$ to induce $a = 0$ and (thus q_H). To see this, note that $D_0 + E_0$ given $\{q_H, a = 0\}$ exceeds $D_0 + E_0$ given $\{q_L, a = 1\}$:

$-cq_H + B_0(\gamma_0)e^{q_H}\int_T^\infty \left(e^{\sigma_X\eta} + \lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta > -cq_L + B_1(\gamma_1)e^{q_L}\int_T^\infty \left(e^{\sigma_X\eta} + \lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta$
$\Leftrightarrow [B_0(\gamma_0)e^{q_H} - B_1(\gamma_1)e^{q_L}]\int_T^\infty \left(e^{\sigma_X\eta} + \lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta > c(q_H - q_L)$. The left-hand side of the preceding inequality exceeds $B_0(\gamma_0)(e^{q_H} - e^{q_L})\int_T^\infty \left(e^{\sigma_X\eta} + \lambda e^{\sigma_X T}\right)\varphi(\eta)d\eta$, which is in turn larger than $B_0(\gamma_0)(e^{q_H} - e^{q_L})G$, which is larger than $c(q_H - q_L)$.

In summary, if $B_0(\gamma_0) \geq B_1(\gamma_1)$, the regulator chooses M_0 and $\frac{M_1}{X}$ to induce $\{q_H, a = 0\}$ if $B_0(\gamma_0)(e^{q_H} - e^{q_L})\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta \geq c(q_H - q_L)$ and chooses M_0 and $\frac{M_1}{X}$ to induce $\{q_L, a = 0\}$ otherwise. ■

PROOF OF PROPOSITION 4

First, we show: $EBX(q_H) > EBX(q_L)$, or equivalently, $\frac{\partial EBX}{\partial q} > 0$.

Proof: By (6),

$$\begin{aligned}\frac{\partial EBX}{\partial q} &= \int_0^{\frac{M_1}{\gamma_0}} \frac{\partial B_1\left(\frac{M_1}{X}\right)X}{\partial q} f(X)dX + \int_{\frac{M_1}{\gamma_0}}^\infty \frac{\partial B_0\left(\frac{M_1}{X}\right)X}{\partial q} f(X)dX \\ &+ \left(\int_0^{\frac{M_1}{\gamma_0}} B_1\left(\frac{M_1}{X}\right)X\frac{\partial f(X)}{\partial q}dX + \int_{\frac{M_1}{\gamma_0}}^\infty B_0\left(\frac{M_1}{X}\right)X\frac{\partial f(X)}{\partial q}dX\right).\end{aligned} \tag{17}$$

(i) $\frac{\partial B_1\left(\frac{M_1}{X}\right)X}{\partial X} = \frac{M_1}{X}\left[\frac{B_1\left(\frac{M_1}{X}\right)}{\frac{M_1}{X}} - B_1'\left(\frac{M_1}{X}\right)\right] > 0$ because $B_1\left(\frac{M_1}{X}\right)$ is concave. In addition, $\frac{\partial X}{\partial q} > 0$. Therefore, the first term is positive.

(ii) $\frac{\partial B_0\left(\frac{M_1}{X}\right)X}{\partial X} = 1 > 0$. In addition, $\frac{\partial X}{\partial q} > 0$. Therefore, the second term is positive.

(iii) The third term is positive because $B_0\left(\frac{M_1}{X}\right)X > B_1\left(\frac{M_1}{X}\right)X$ and a higher value of q shifts the distribution of $X \equiv e^{q+\sigma_X\eta}$ to the right.

The bank chooses q to maximize $E_0 = -cq + \mathbb{E}[\mathbf{1}_{D_1+E_1\geq M_0} \bullet (D_1 + E_1 - M_0)] = -cq + \mathbb{E}[\mathbf{1}_{EBX\geq M_0} \bullet (EBX - M_0)]$.

(i) The case of $M_0 \leq EBX(q_L)$.

If the bank chooses q_H, $E_0(q_H) = -cq_H + EBX(q_H) - M_0$. If the bank chooses q_L, $E_0(q_L) = -cq_L + EBX(q_L) - M_0$. Therefore, the bank chooses q_H if and only if $EBX(q_H) - EBX(q_L) \geq c(q_H - q_L)$, which holds by assumption.

(ii) The case of $M_0 \in (EBX(q_L), EBX(q_H)]$.

If the bank chooses q_H, $E_0(q_H) = -cq_H + EBX(q_H) - M_0$. If the bank chooses q_L, $E_0(q_L) = -cq_L$. Therefore, the bank chooses q_H if and only if $M_0 \leq EBX(q_H) - c(q_H - q_L)$.

(iii) The case of $M_0 > EBX(q_H)$.

If the bank chooses q_H, $E_0(q_H) = -cq_H$. If the bank chooses q_L, $E_0(q_L) = -cq_L$. Therefore, the bank chooses q_L.

In summary, (1) when $M_0 \leq EBX(q_H) - c(q_H - q_L)$, the bank chooses q_H, and $E_0(q_H) = -cq_H + EBX(q_H) - M_0$ and $D_0 = (1+\lambda)M_0$, and thus $D_0 + E_0 = -cq_H + EBX(q_H) + \lambda M_0$, and (2) otherwise, the bank chooses q_L, and $E_0(q_L) = -cq_L$ and $D_0 = 0$, and thus $D_0 + E_0 = -cq_L$, where D_0 is derived from $D_0 = \mathbb{E}[\mathbf{1}_{EBX \geq M_0} \bullet (1+\lambda)M_0]$. ■

PROOF OF PROPOSITION 5

From the proof of Proposition 4, two cases exist:

(i) If $M_0 \leq EBX(q_H) - c(q_H - q_L)$, $D_0 + E_0 = -cq_H + EBX(q_H) + \lambda M_0$. Thus, the optimal value of M_0 is $EBX(q_H) - c(q_H - q_L)$, which implies that $D_0 + E_0 = -cq_H + (1+\lambda)EBX(q_H) - \lambda c(q_H - q_L)$.

(ii) If $M_0 > EBX(q_H) - c(q_H - q_L)$, $D_0 + E_0 = -cq_L$.

Thus, $D_0 + E_0$ in case (i) exceeds that in case (ii) if and only if $EBX(q_H) > c(q_H - q_L)$, which is implied by the assumption that $EBX(q_H) - EBX(q_L) \geq c(q_H - q_L)$.

Therefore, the global solution is $M_0 = EBX(q_H) - c(q_H - q_L)$.

The first-order condition of $D_0 + E_0$ in case (i) with respect to M_1 characterizes the optimal solution for M_1: $\frac{\partial(D_0+E_0)}{\partial M_1} = (1+\lambda)\frac{\partial EBX(q_H)}{\partial M_1} = 0$, which is equivalent to (7). ■

LEMMA 2 (Comparative Statics)

(i) $\frac{\partial \gamma_0}{\partial \sigma_Z} < 0$ and $\frac{\partial \gamma_0}{\partial k} > 0$.

(ii) $\frac{\partial B_0(\gamma_0)}{\partial \sigma_Z} < 0$, $\frac{\partial B_0(\gamma_0)}{\partial k} > 0$, and $\frac{\partial B_0(\gamma_0)}{\partial \lambda} > 0$.

(iii) $\frac{\partial \gamma_1}{\partial \sigma_Z} > 0$, $\frac{\partial \gamma_1}{\partial k} < 0$, $\frac{\partial \gamma_1}{\partial \alpha} > 0$, and $\frac{\partial \gamma_1}{\partial \lambda} > 0$.

(iv) $\frac{\partial B_1(\gamma_1)}{\partial \sigma_Z} > 0$ for $\sigma_Z \geq \sqrt{\frac{2}{\pi}} / \left(\frac{\lambda}{(1+\lambda)(1-\alpha)}\right)$ and $\alpha < 1/(1+\lambda)$, $\frac{\partial B_1(\gamma_1)}{\partial k} < 0$, $\frac{\partial B_1(\gamma_1)}{\partial \alpha} > 0$, and $\frac{\partial B_1(\gamma_1)}{\partial \lambda} > 0$.

PROOF OF LEMMA 2

(i) (9) implies that $\frac{\partial \gamma_0}{\partial \sigma_Z} < 0$ and $\frac{\partial \gamma_0}{\partial k} > 0$.

Proof of $\frac{\partial \gamma_0}{\partial \sigma_Z} < 0$: Differentiating the right-hand side of (9) with respect to σ_Z yields $\int_{(k+ln\gamma_0)/\sigma_Z}^{\infty} e^{\sigma_Z \eta - k} \eta \varphi(\eta) d\eta$. If it is positive, then $\frac{\partial \gamma_0}{\partial \sigma_Z} < 0$.

Show: $\int_{(k+ln\gamma_0)/\sigma_Z}^{\infty} e^{\sigma_Z\eta-k}\eta\varphi(\eta)d\eta > 0$. Proof: Define $p(\eta) \equiv e^{\sigma_Z\eta-k}\eta\varphi(\eta)$. For any $b > 0$, $|p(b)| = e^{\sigma_Z b-k}b\varphi(b) > |p(-b)| = e^{-\sigma_Z b-k}b\varphi(-b)$.

Proof of $\frac{\partial\gamma_0}{\partial k} > 0$: The right-hand side of (9) with respect to k is decreasing in k, which implies that $\frac{\partial\gamma_0}{\partial k} > 0$.

(ii) Evaluating $B_0\left(\frac{M_1}{X}\right)$ in (1) at $\frac{M_1}{X} = \gamma_0$ yields

$$B_0\left(\gamma_0\right) \equiv 1 + \lambda\gamma_0; \tag{18}$$

Differentiating (18) with respect to σ_Z, k, and λ yields the desired results.

(iii) (4) implies that $\frac{\partial\gamma_1}{\partial\sigma_Z} > 0$, $\frac{\partial\gamma_1}{\partial k} < 0$, $\frac{\partial\gamma_1}{\partial\alpha} > 0$, and $\frac{\partial\gamma_1}{\partial\lambda} > 0$.

(iv) Evaluating $B_1\left(\frac{M_1}{X}\right)$ in (2) at $\frac{M_1}{X} = \gamma_1$ yields

$$\begin{aligned} B_1\left(\gamma_1\right) \equiv & \int_{(k+ln\gamma_1)/\sigma_Z}^{\infty}\left(e^{\sigma_Z\eta-k} + \lambda\gamma_1\right)\varphi(\eta)d\eta \\ & + \int_{-\infty}^{(k+ln\gamma_1)/\sigma_Z}(1+\lambda)\alpha e^{\sigma_Z\eta-k}\varphi(\eta)d\eta. \end{aligned} \tag{19}$$

Show: $\frac{\partial B_1(\gamma_1)}{\partial\sigma_Z} > 0$ for $\sigma_Z \geq \sqrt{\frac{2}{\pi}}/\left(\frac{\lambda}{(1+\lambda)(1-\alpha)}\right)$ and $\alpha < 1/(1+\lambda)$.

$\frac{\partial B_1(\gamma_1)}{\partial\sigma_Z} = \left[\int_{(k+ln\gamma_1)/\sigma_Z}^{\infty} e^{\sigma_Z\eta-k}\eta\varphi(\eta)d\eta + \int_{-\infty}^{(k+ln\gamma_1)/\sigma_Z}(1+\lambda)\alpha e^{\sigma_Z\eta-k}\eta\varphi(\eta)d\eta\right] + (1+\lambda)(1-\alpha)\gamma_1\left(k+ln\gamma_1\right)/\sigma_Z\varphi\left(\left(k+ln\gamma_1\right)/\sigma_Z\right)/\sigma_Z$.

To show the bracketed term is positive, first define $p(\eta) \equiv e^{\sigma_Z\eta-k}\eta\varphi(\eta)$. For any $b > 0$, $|p(b)| = e^{\sigma_Z b-k}b\varphi(b) > |p(-b)| = e^{-\sigma_Z b-k}b\varphi(-b)$.

Therefore, for $\eta \in [-\left(k+ln\gamma_1\right)/\sigma_Z, \left(k+ln\gamma_1\right)/\sigma_Z]$, $|(1+\lambda)\alpha p(b)| > |(1+\lambda)\alpha p(-b)|$. In addition, for any $b > \left(k+ln\gamma_1\right)/\sigma_Z$, $|p(b)| > |(1+\lambda)\alpha p(-b)|$ by the assumption $(1+\lambda)\alpha < 1$.

The last term is nonnegative because $k+ln\gamma_1 \geq 0$. To see this, note that $h\left(\left(k+ln\gamma_1\right)/\sigma_Z\right) = \sigma_Z\times\frac{\lambda}{(1+\lambda)(1-\alpha)}$, which is greater than $h(0) = \frac{\varphi(0)}{1-\Phi(0)} = \sqrt{\frac{2}{\pi}}$ if and only if $\sigma_Z \geq \sqrt{\frac{2}{\pi}}/\left(\frac{\lambda}{(1+\lambda)(1-\alpha)}\right)$, which is satisfied by assumption.

Show: $\frac{\partial B_1(\gamma_1)}{\partial k} < 0$.

$\frac{\partial B_1(\gamma_1)}{\partial k} = -\int_{(k+ln\gamma_1)/\sigma_Z}^{\infty} e^{\sigma_Z\eta-k}\varphi(\eta)d\eta - \int_{-\infty}^{(k+ln\gamma_1)/\sigma_Z}(1+\lambda)\alpha e^{\sigma_Z\eta-k}\varphi(\eta)d\eta - (1+\lambda)(1-\alpha)\gamma_1\varphi\left(\left(k+ln\gamma_1\right)/\sigma_Z\right)/\sigma_Z < 0$.

Differentiating (19) with respect to α and λ respectively yields $\frac{\partial\gamma_1}{\partial\alpha} > 0$ and $\frac{\partial\gamma_1}{\partial\lambda} > 0$. ■

LEMMA 3 (Comparative Statics): $B_1(\gamma_1) > B_0(\gamma_0)$ if and only if

(i) σ_Z is sufficiently high; (ii) k is sufficiently low; (iii) α is sufficiently high; and (iv) λ is sufficiently high.

PROOF OF LEMMA 3

Result (i) is implied by $\frac{\partial B_0(\gamma_0)}{\partial \sigma_Z} < 0$ and $\frac{\partial B_1(\gamma_1)}{\partial \sigma_Z} > 0$ in Lemma 2.

Result (ii) is implied by $\frac{\partial B_0(\gamma_0)}{\partial k} > 0$ and $\frac{\partial B_1(\gamma_1)}{\partial k} < 0$ in Lemma 2.

Result (iii) is implied by $\frac{\partial B_0(\gamma_0)}{\partial \alpha} = 0$ and $\frac{\partial B_1(\gamma_1)}{\partial \alpha} > 0$ in Lemma 2.

To prove result (iv), note that by Lemma 2, $\frac{\partial B_0(\gamma_0)}{\partial \lambda} = \gamma_0 > 0$ and

$$\frac{\partial B_1(\gamma_1)}{\partial \lambda} = \int_{(k+ln\gamma_1)/\sigma_Z}^{\infty} \gamma_1 \varphi(\eta) d\eta + \int_{-\infty}^{(k+ln\gamma_1)/\sigma_Z} \alpha e^{\sigma_Z \eta - k} \varphi(\eta) d\eta > 0. \tag{20}$$

Furthermore, $\frac{\partial^2 B_1(\gamma_1)}{\partial \lambda^2} = [1 - \Phi((k + ln\gamma_1)/\sigma_Z)] [1 - (1-\alpha) h((k + ln\gamma_1)/\sigma_Z)/\sigma_Z] \frac{\partial \gamma_1}{\partial \lambda} = [1 - \Phi((k + ln\gamma_1)/\sigma_Z)] \left[1 - \frac{\lambda}{1+\lambda}\right] \frac{\partial \gamma_1}{\partial \lambda} > 0$. Therefore, $B_1(\gamma_1)$ is an increasing and convex function of λ.

At $\lambda = 0$, $(k + ln\gamma_1)/\sigma_Z = -\infty$ and thus $B_1(\gamma_1) = \int_{-\infty}^{\infty} e^{\sigma_Z \eta - k} \varphi(\eta) d\eta = \mathbb{E}[Z] = e^{\frac{1}{2}\sigma_Z^2 - k}$, which is less than $B_0(\gamma_0) = 1$ because $\frac{1}{2}\sigma_Z^2 < k$ by assumption. ■

LEMMA 4 (Comparative Statics):

(i) $\frac{\partial T}{\partial \sigma_X} > 0$ and $\frac{\partial T}{\partial \lambda} > 0$.

(ii) $B_1(\gamma_1) \frac{e^{q_H} - e^{q_L}}{q_H - q_L} \int_T^{\infty} e^{\sigma_X \eta} \varphi(\eta) d\eta \geq c$ is more likely to hold for sufficiently high values of σ_Z; sufficiently low values of k; sufficiently high values of α; and sufficiently high values of λ.

(iii) $B_0(\gamma_0) \frac{e^{q_H} - e^{q_L}}{q_H - q_L} \int_T^{\infty} e^{\sigma_X \eta} \varphi(\eta) d\eta \geq c$ is more likely to hold for sufficiently high values of σ_Z; sufficiently low values of k; and sufficiently low values of λ.

PROOF OF LEMMA 4

Result (i) is implied by equation (5) that defines T.

The results regarding σ_Z, k, α in (ii) and (iii) are implied directly by Lemma 2, which shows that $\frac{\partial B_0(\gamma_0)}{\partial \sigma_Z} < 0$, $\frac{\partial B_1(\gamma_1)}{\partial \sigma_Z} > 0$, $\frac{\partial B_0(\gamma_0)}{\partial k} > 0$, $\frac{\partial B_1(\gamma_1)}{\partial k} < 0$, $\frac{\partial B_1(\gamma_1)}{\partial \alpha} > 0$.

Lemma 2 also shows that $\frac{\partial B_0(\gamma_0)}{\partial \lambda} > 0$ and $\frac{\partial B_1(\gamma_1)}{\partial \lambda} > 0$. In addition, the derivative of $\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta$ with respect to λ is $-e^{\sigma_X T}\varphi(T)\frac{\partial T}{\partial \lambda} < 0$. ■

PROOF OF PROPOSITION 6

By Lemma 2, $\frac{\partial B_0(\gamma_0)}{\partial \sigma_Z} < 0$ and $\frac{\partial B_1(\gamma_1)}{\partial \sigma_Z} > 0$. Define σ_Z^{II} as the value of σ_Z where $B_0(\gamma_0)$ and $B_1(\gamma_1)$ intersect. In addition, Define σ_Z^I as the value of σ_Z where $B_0(\gamma_0)$ and $\frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$ intersect and σ_Z^{III} as the value of σ_Z where $B_1(\gamma_1)$ and $\frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$ intersect. Thus, $\sigma_Z^I < \sigma_Z^{II} < \sigma_Z^{III}$.

For $\sigma_Z \leq \sigma_Z^I$, because $B_0(\gamma_0) > B_1(\gamma_1)$, $a^{FV} = 0$, and because $B_0(\gamma_0) > \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$, $q^{FV} = q_H$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_0 X$and $M_0^{FV} = e^{q_H+\sigma_X T}B_0(\gamma_0)$.

For $\sigma_Z \in (\sigma_Z^I, \sigma_Z^{II}]$, because $B_0(\gamma_0) > B_1(\gamma_1)$, $a^{FV} = 0$, and because $B_0(\gamma_0) < \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$, $q^{FV} = q_L$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_0 X$and $M_0^{FV} = e^{q_L+\sigma_X T}B_0(\gamma_0)$.

For $\sigma_Z \in (\sigma_Z^{II}, \sigma_Z^{III}]$, because $B_1(\gamma_1) > B_0(\gamma_0)$, $a^{FV} = 1$, and because $B_1(\gamma_1) < \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$, $q^{FV} = q_L$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_1 X$and $M_0^{FV} = e^{q_L+\sigma_X T}B_1(\gamma_1)$.

For $\sigma_Z > \sigma_Z^{III}$, because $B_1(\gamma_1) > B_0(\gamma_0)$, $a^{FV} = 1$, and because $B_1(\gamma_1) > \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$, $q^{FV} = q_H$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_1 X$and $M_0^{FV} = e^{q_H+\sigma_X T}B_1(\gamma_1)$. ■

LEMMA 5 (Comparative Statics):

(i) Define L as the left-hand side of (7). The sign of $\frac{\partial L}{\partial p}$ is the same as the sign of $\frac{\partial M_1^{HC}}{\partial p}$ where p is a generic symbol for parameters and $p \in \{\sigma_Z, k, \alpha, \sigma_X, \lambda\}$.

(ii) $Pr(a^{HC} = 1) = \Phi\left(\left(ln\frac{M_1^{HC}}{\gamma_0} - q_H\right)/\sigma_X\right)$.

PROOF OF LEMMA 5

(i) Define L as the left-hand side of (7). $\frac{\partial L}{\partial M_1^{HC}} < 0$ and $\frac{\partial L}{\partial \gamma_0} = 0$.

I can rewrite L as

$$L \equiv \int_0^{\frac{M_1^{HC}}{\gamma_0}} \left[1 - \Phi\left(\left(k + ln\frac{M_1^{HC}}{X}\right)/\sigma_Z\right)\right]$$
$$\left[\lambda - (1+\lambda)(1-\alpha)h\left(\left(k + ln\frac{M_1^{HC}}{X}\right)/\sigma_Z\right)/\sigma_Z\right] f(X)dX \qquad (21)$$
$$+ \int_{\frac{M_1^{HC}}{\gamma_0}}^{\infty} \lambda f(X)dX$$

Because $\frac{\partial L}{\partial M_1^{HC}}\frac{\partial M_1^{HC}}{\partial p} + \frac{\partial L}{\partial p} = 0$ and $\frac{\partial L}{\partial M_1^{HC}} < 0$, the sign of $\frac{\partial L}{\partial p}$ is the same as the sign of $\frac{\partial M_1^{HC}}{\partial p}$.

(ii) By Proposition 5, $a^{HC} = 1$ if $X \equiv e^{q_H + \sigma_X \eta} < \frac{M_1^{HC}}{\gamma_0}$. Thus, $Pr(a^{HC} = 1) = \Phi\left(\left(ln\frac{M_1^{HC}}{\gamma_0} - q_H\right)/\sigma_X\right)$. ■

PROOF OF PROPOSITION 7

By (6), $\frac{\partial EBX}{\partial \gamma_0} = 0$. $\frac{\partial M_0^{HC}}{\partial \sigma_Z} = \frac{\partial EBX}{\partial \sigma_Z} = \int_0^{\frac{M_1^{HC}}{\gamma_0}} \frac{\partial B_1\left(\frac{M_1^{HC}}{X}\right)}{\partial \sigma_Z} X f(X)dX > 0$. By (21), $\frac{\partial L}{\partial \sigma_Z} > 0$ and thus by Lemma 5, $\frac{\partial M_1^{HC}}{\partial \sigma_Z} > 0$. Because $\frac{\partial M_1^{HC}}{\partial \sigma_Z} > 0$ and $\frac{\partial \gamma_0}{\partial \sigma_Z} < 0$, by Lemma 5, $\frac{\partial Pr(a^{HC}=1)}{\partial \sigma_Z} > 0$. ■

PROOF OF PROPOSITION 8

From the proof of Proposition 5,

$$D_0^{HC} + E_0^{HC} = -cq_H + EBX(q_H, M_1^{HC}) + \lambda M_0^{HC}. \qquad (22)$$

From the proof of Proposition 3,

$$D_0^{FV} + E_0^{FV} = -cq + \int_T^{\infty} \left(B_a(\gamma_a)X + \lambda M_0^{FV}\right)\varphi(\eta)d\eta. \qquad (23)$$

Thus, $\frac{\partial(D_0^{HC}+E_0^{HC})}{\partial \sigma_Z} = (1+\lambda)\frac{\partial EBX(q_H, M_1^{HC})}{\partial \sigma_Z} > 0$, $\frac{\partial(D_0^{FV}+E_0^{FV})}{\partial \sigma_Z} < 0$ when $a = 0$ and $\frac{\partial(D_0^{FV}+E_0^{FV})}{\partial \sigma_Z} > 0$ when $a = 1$. These facts imply the statement of Proposition 8. ■

PROOF OF PROPOSITION 9

In the historical cost regime, by (6), $\frac{\partial EBX}{\partial \gamma_0} = 0$. $\frac{\partial M_0^{HC}}{\partial k} = \frac{\partial EBX}{\partial k} = \int_0^{\frac{M_1^{HC}}{\gamma_0}} \frac{\partial B_1\left(\frac{M_1^{HC}}{X}\right)}{\partial k} X f(X) dX < 0$. By (21), $\frac{\partial L}{\partial k} < 0$ and thus $\frac{\partial M_1^{HC}}{\partial k} < 0$. Because $\frac{\partial M_1^{HC}}{\partial k} < 0$ and $\frac{\partial \gamma_0}{\partial k} > 0$, $\frac{\partial Pr(a^{HC}=1)}{\partial k} < 0$. By (22), $\frac{\partial (D_0^{HC}+E_0^{HC})}{\partial k} = (1+\lambda)\frac{\partial EBX(q_H, M_1^{HC})}{\partial k} < 0$.

In the fair value regime, by (23), $\frac{\partial (D_0^{FV}+E_0^{FV})}{\partial k} > 0$ when $a = 0$ and $\frac{\partial (D_0^{FV}+E_0^{FV})}{\partial k} < 0$ when $a = 1$.

Because k works against σ_Z, the rest of the proof follows the same logic in the proof of Proposition 8 and thus is omitted. ■

PROOF OF PROPOSITION 10

In the fair value regime, by Lemma 2, $\frac{\partial B_0(\gamma_0)}{\partial \alpha} = 0$ and $\frac{\partial B_1(\gamma_1)}{\partial \alpha} > 0$. Define α^{II} as the value of α where $B_0(\gamma_0)$ and $B_1(\gamma_1)$ intersect. In addition, Define α^{III} as the value of α where $B_1(\gamma_1)$ and $\frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$ intersect. Thus, $\sigma_Z^{II} < \sigma_Z^{III}$.

For $\sigma_Z \le \sigma_Z^{II}$, because $B_0(\gamma_0) > B_1(\gamma_1)$, $a^{FV} = 0$, and because $B_0(\gamma_0) < \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$, $q^{FV} = q_L$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_0 X$and $M_0^{FV} = e^{q_L+\sigma_X T}B_0(\gamma_0)$.

For $\sigma_Z \in (\sigma_Z^{II}, \sigma_Z^{III}]$, because $B_1(\gamma_1) > B_0(\gamma_0)$, $a^{FV} = 1$, and because $B_1(\gamma_1) < \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$, $q^{FV} = q_L$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_1 X$and $M_0^{FV} = e^{q_L+\sigma_X T}B_1(\gamma_1)$.

For $\sigma_Z > \sigma_Z^{III}$, because $B_1(\gamma_1) > B_0(\gamma_0)$, $a^{FV} = 1$, and because $B_1(\gamma_1) > \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}$, $q^{FV} = q_H$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_1 X$and $M_0^{FV} = e^{q_H+\sigma_X T}B_1(\gamma_1)$.

$\frac{\partial (D_0^{FV}+E_0^{FV})}{\partial \alpha} = 0$ when $a = 0$ and $\frac{\partial (D_0^{FV}+E_0^{FV})}{\partial \alpha} > 0$ when $a = 1$.

In the historical cost regime, by (6), $\frac{\partial M_0^{HC}}{\partial \alpha} = \frac{\partial EBX}{\partial \alpha} = \int_0^{\frac{M_1^{HC}}{\gamma_0}} \frac{\partial B_1\left(\frac{M_1^{HC}}{X}\right)}{\partial \alpha} X f(X) dX > 0$.

By (21), $\frac{\partial L}{\partial \alpha} > 0$ and thus $\frac{\partial M_1^{HC}}{\partial \alpha} > 0$.

$\frac{\partial (D_0^{HC}+E_0^{HC})}{\partial \alpha} = (1+\lambda)\frac{\partial EBX(q_H, M_1^{HC})}{\partial \alpha} > 0$.

Because $\frac{\partial M_1^{HC}}{\partial \alpha} > 0$, $\frac{\partial Pr(a^{HC}=1)}{\partial \alpha} > 0$. ■

PROOF OF PROPOSITION 11

Lemma 4 states that the bank chooses q_H if and only if $B_a(\gamma_a)\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta \ge c$.

Because $\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta$ is increasing in σ_X, a higher value of σ_X implies that it is more likely that q_H will be chosen.

Furthermore, Lemma 4 states that M_0^* is increasing in $\sigma_X T$, which in turn is increasing in σ_X because $T \geq 0$.

Show: $\int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta$ is increasing in σ_X.

Define $A \equiv \int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta = \mathbb{E}[e^{\sigma_X \eta} | e^{\sigma_X \eta} \geq e^{\sigma_X T}]$. Because $e^{\sigma_X \eta}$ follows the lognormal distribution, $A = e^{\frac{1}{2}\sigma_X^2} \cdot \frac{\Phi(\sigma_X - T)}{1-\Phi(T)}$. Therefore, $\frac{\partial A}{\partial \sigma_X} > 0$ if $0 < \frac{\partial T}{\partial \sigma_X} < 1$.

By (5), $\frac{\partial T}{\partial \sigma_X} = \frac{\lambda}{1+\lambda}/h'(T) > 0$ because the hazard rate for the normal distribution is increasing.

Moreover, $\frac{\partial T}{\partial \sigma_X} < 1$ if and only if $h'(T) > \frac{\lambda}{1+\lambda}$. Because the hazard rate for the normal distribution is convex, $h'(T) > \frac{\lambda}{1+\lambda}$ holds for $T > T^*$ where $h'(T^*) = \frac{\lambda}{1+\lambda}$. Because $\frac{\partial T}{\partial \sigma_X} > 0$, $T > T^*$ is equivalent to $\sigma_X > \sigma_X^*$ where $\sigma_X^* \equiv h(T^*)/\left(\frac{\lambda}{1+\lambda}\right)$. This last inequality is satisfied by the assumption $\sigma_X \geq max\{h(T^*), \sqrt{\frac{2}{\pi}}\}/\left(\frac{\lambda}{1+\lambda}\right)$.

Show: $T \geq 0$.

By (5), $h(T) = \sigma_X \cdot \frac{\lambda}{1+\lambda}$. Moreover, $h(0) = \sqrt{\frac{2}{\pi}}$. Therefore, $T \geq 0$ if and only if $\sigma_X \cdot \frac{\lambda}{1+\lambda} \geq \sqrt{\frac{2}{\pi}}$, which is satisfied by the assumption $\sigma_X \geq max\{h(T^*), \sqrt{\frac{2}{\pi}}\}/\left(\frac{\lambda}{1+\lambda}\right)$.

Show: $\frac{\partial(D_0^{FV}+E_0^{FV})}{\partial \sigma_X} > 0$.

$\frac{\partial(D_0^{FV}+E_0^{FV})}{\partial \sigma_X} = B_a(\gamma_a)e^q\left[\frac{\partial \int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}{\partial \sigma_X} + \lambda \frac{\partial e^{\sigma_X T}[1-\Phi(T)]}{\partial \sigma_X}\right]$. $\frac{\partial \int_T^\infty e^{\sigma_X \eta}\varphi(\eta)d\eta}{\partial \sigma_X} > 0$. $\frac{\partial e^{\sigma_X T}[1-\Phi(T)]}{\partial \sigma_X} = e^{\sigma_X T}[1-\Phi(T)]\{T + [\sigma_X - h(T)]\frac{\partial T}{\partial \sigma_X}\} > 0$ if $\sigma_X > h(T)$.

Because $\frac{\partial h(T)}{\partial \sigma_X} = h'(T)\frac{\partial T}{\partial \sigma_X} = \frac{\lambda}{1+\lambda} < 1$, $\sigma_X > h(T)$ iff $\sigma_X > \sigma_X^* \equiv h(T^*)/\left(\frac{\lambda}{1+\lambda}\right)$.

Because $B_0\left(\frac{M_1}{X}\right) > B_1\left(\frac{M_1}{X}\right)$ and a higher value of σ_X leads to a higher degree of skewness, $\frac{\partial M_0^{HC}}{\partial \sigma_X} = \frac{\partial EBX}{\partial \sigma_X} < 0$.

Because $\lambda > B_1'\left(\frac{M_1}{X}\right)$ and a higher value of σ_X leads to a higher degree of skewness, $\frac{\partial L}{\partial \sigma_X} < 0$ and thus $\frac{\partial M_1^{HC}}{\partial \sigma_X} < 0$.

$\frac{\partial(D_0^{HC}+E_0^{HC})}{\partial \sigma_X} = (1+\lambda)\frac{\partial EBX(q_H, M_1^{HC})}{\partial \sigma_X} < 0$.

Because $\frac{\partial M_1^{HC}}{\partial \sigma_X} < 0$, $\frac{\partial Pr(a^{HC}=1)}{\partial \sigma_X} < 0$. ■

PROOF OF PROPOSITION 12

Proof of part (i):

By Lemma 2, $\frac{\partial B_0(\gamma_0)}{\partial\lambda} > 0$ and $\frac{\partial B_1(\gamma_1)}{\partial\lambda} > 0$. By Lemma 4, $\frac{\partial}{\partial\lambda}\left(\frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta}\right) > 0$. Define λ^{II} as the value of λ where $B_0(\gamma_0)$ and $B_1(\gamma_1)$ intersect. In addition, Define λ^I as the value of λ where $B_0(\gamma_0)$ and $\frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta}$ intersect and λ^{III} as the value of λ where $B_1(\gamma_1)$ and $\frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta}$ intersect. Thus, $\lambda^I < \lambda^{II} < \lambda^{III}$.

For $\lambda \le \lambda^I$, because $B_0(\gamma_0) > B_1(\gamma_1)$, $a^{FV} = 0$, and because $B_0(\gamma_0) > \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta}$, $q^{FV} = q_H$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_0 X$and $M_0^{FV} = e^{q_H+\sigma_X T}B_0(\gamma_0)$.

For $\lambda \in (\lambda^I, \lambda^{II}]$, because $B_0(\gamma_0) > B_1(\gamma_1)$, $a^{FV} = 0$, and because $B_0(\gamma_0) < \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta}$, $q^{FV} = q_L$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_0 X$and $M_0^{FV} = e^{q_L+\sigma_X T}B_0(\gamma_0)$.

For $\lambda \in (\lambda^{II}, \lambda^{III}]$, because $B_1(\gamma_1) > B_0(\gamma_0)$, $a^{FV} = 1$, and because $B_1(\gamma_1) < \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta}$, $q^{FV} = q_L$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_1 X$and $M_0^{FV} = e^{q_L+\sigma_X T}B_1(\gamma_1)$.

For $\lambda > \lambda^{III}$, because $B_1(\gamma_1) > B_0(\gamma_0)$, $a^{FV} = 1$, and because $B_1(\gamma_1) > \frac{c}{\frac{e^{q_H}-e^{q_L}}{q_H-q_L}\int_T^\infty e^{\sigma_X\eta}\varphi(\eta)d\eta}$, $q^{FV} = q_H$. Therefore, by Proposition 5, the regulator chooses $M_1^{FV} = \gamma_1 X$and $M_0^{FV} = e^{q_H+\sigma_X T}B_1(\gamma_1)$.

Proof of part (ii):

By (6), $\frac{\partial M_0^{HC}}{\partial\lambda} = \frac{\partial EBX}{\partial\lambda} = \int_0^{\frac{M_1^{HC}}{\gamma_0}} \frac{\partial B_1\left(\frac{M_1^{HC}}{X}\right)}{\partial\lambda} Xf(X)dX + \int_{\frac{M_1^{HC}}{\gamma_0}}^\infty \frac{\partial B_0\left(\frac{M_1^{HC}}{X}\right)}{\partial\lambda} Xf(X)dX > 0$.

Show: $\frac{\partial L}{\partial\lambda} > 0$ and thus $\frac{\partial M_1^{HC}}{\partial\lambda} > 0$.

By (21), $\frac{\partial L}{\partial\lambda} = \int_{-\infty}^{\left(ln\frac{M_1^{HC}}{\gamma_0}-q_H\right)/\sigma_X} S(\eta)\varphi(\eta)d\eta + \int_{\left(ln\frac{M_1^{HC}}{\gamma_0}-q_H\right)/\sigma_X}^\infty \varphi(\eta)d\eta$ where $S(\eta) \equiv \left[1-\Phi\left(\left(k+ln\frac{M_1^{HC}}{X}\right)/\sigma_Z\right)\right]\left[1-(1-\alpha)h\left(\left(k+ln\frac{M_1^{HC}}{X}\right)/\sigma_Z\right)/\sigma_Z\right]$.

Case (i): $\left(ln\frac{M_1^{HC}}{\gamma_0} - q_H\right)/\sigma_X > 0$, and then the value of η can be divided into four regions:

Region I ($\eta < -\left(ln\frac{M_1^{HC}}{\gamma_0} - q_H\right)/\sigma_X$); Region II ($\eta \in [-\left(ln\frac{M_1^{HC}}{\gamma_0} - q_H\right)/\sigma_X, 0]$); Region III ($\eta \in [0, \left(ln\frac{M_1^{HC}}{\gamma_0} - q_H\right)/\sigma_X]$); and Region IV ($\eta > \left(ln\frac{M_1^{HC}}{\gamma_0} - q_H\right)/\sigma_X$)).

Because $1 > S(\eta)$, the value of the integrand in Region IV exceeds the absolute value of the integrand in Region I.

Because $S'(\eta) > 0$, the value of the integrand in Region III exceeds the absolute value of the integrand in Region II.

$\frac{\partial(D_0^{FV}+E_0^{FV})}{\partial\lambda} > 0.$

Because $\frac{\partial M_1^{HC}}{\partial\lambda} > 0$, $\frac{\partial Pr(a^{HC}=1)}{\partial\lambda} > 0$. $\frac{\partial(D_0^{HC}+E_0^{HC})}{\partial\lambda} = (1+\lambda)\frac{\partial EBX(q_H,M_1^{HC})}{\partial\lambda} + EBX(q_H, M_1^{HC}) - c(q_H - q_L) > 0$. ■

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